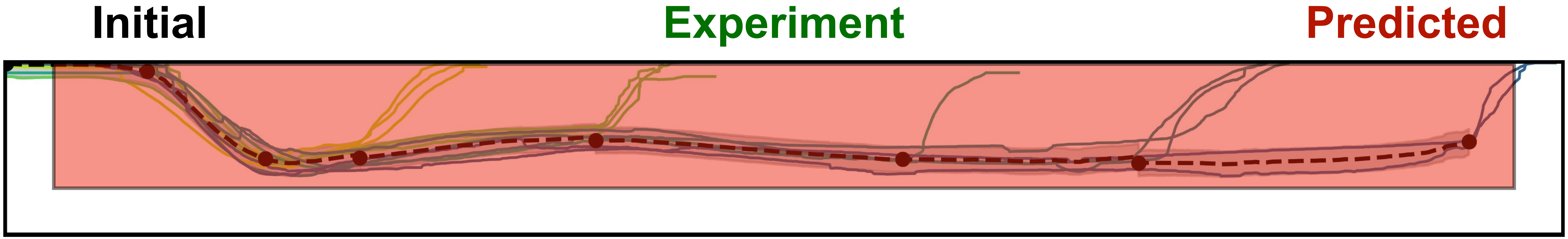
Can we predict iceberg melting?



Model underestimates melting and ignores geometry!

The Big Questions

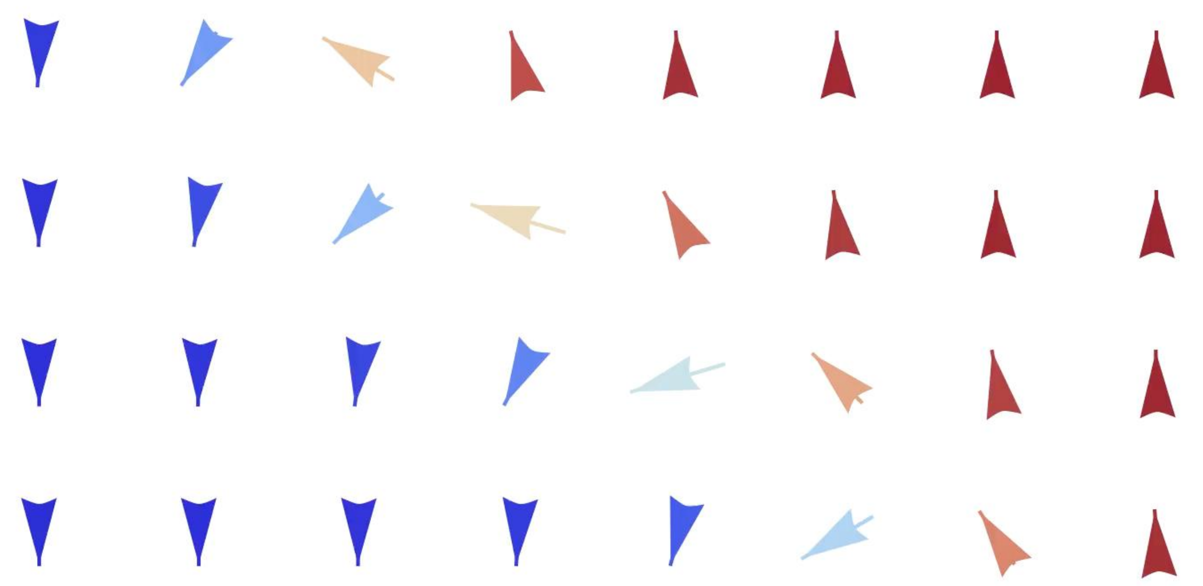
1. Can we build a better model?

2. Can we simulate it efficiently?

3. What does it tell us?

The model hierarchy

Atomistic Models

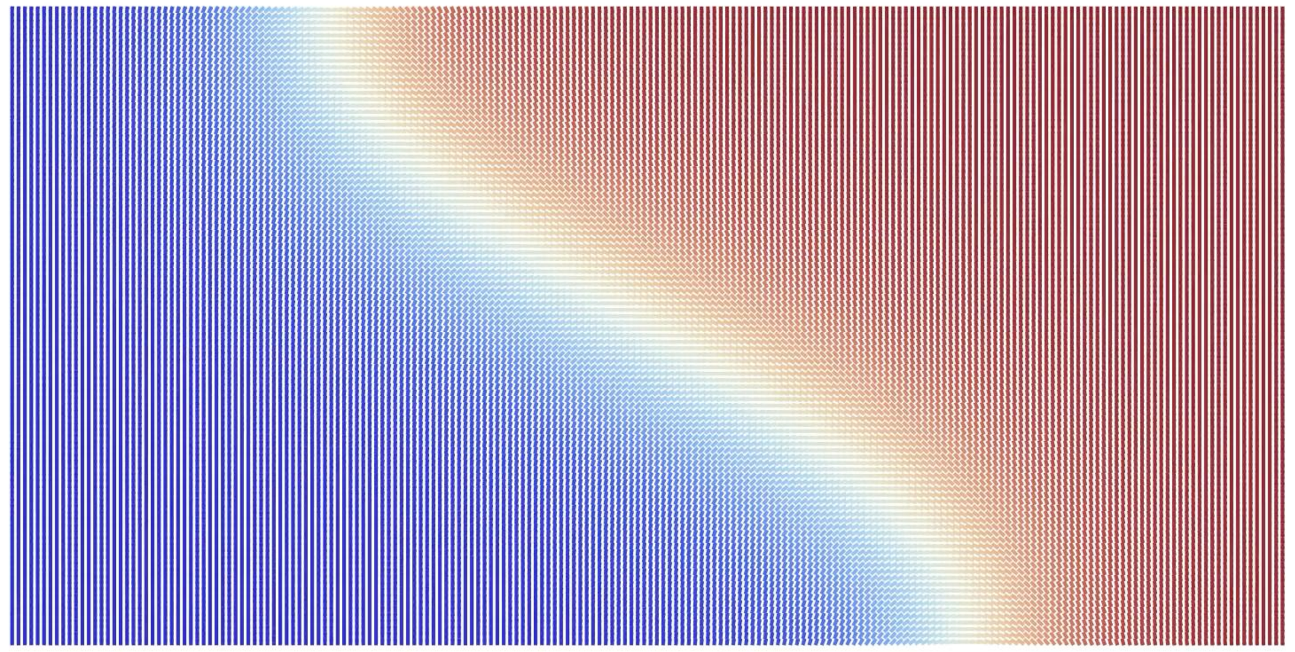


$n \to \infty$

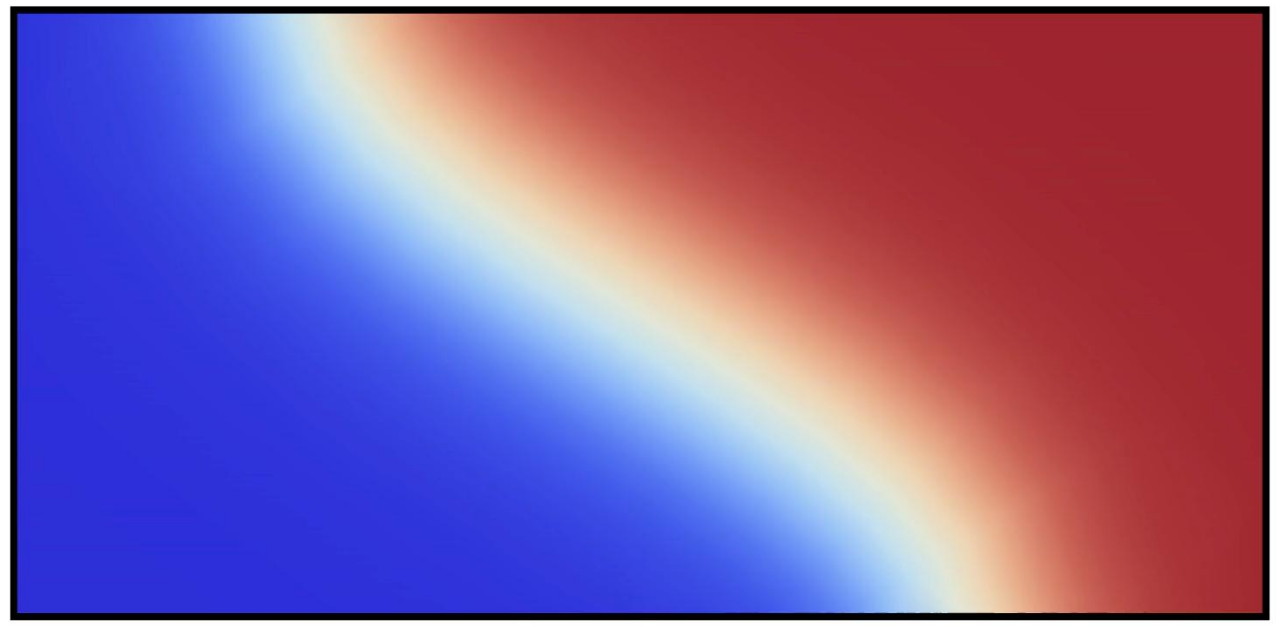
The model hierarchy

Atomistic Models

Phase-field Models



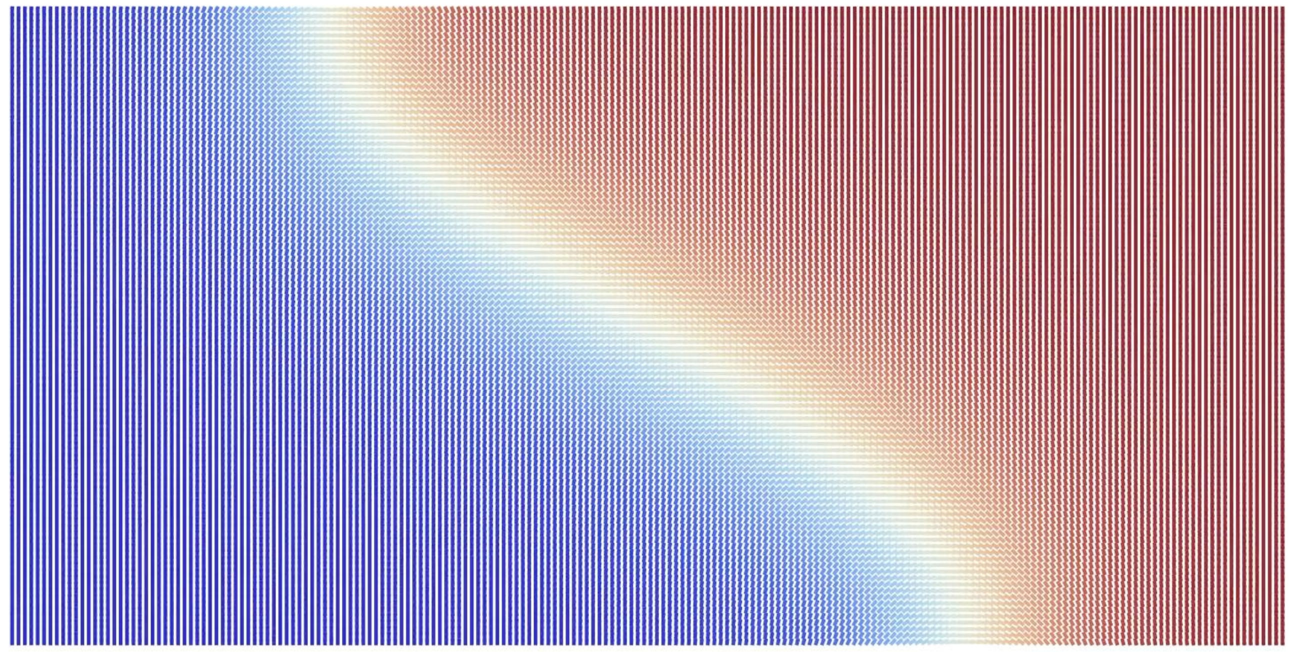
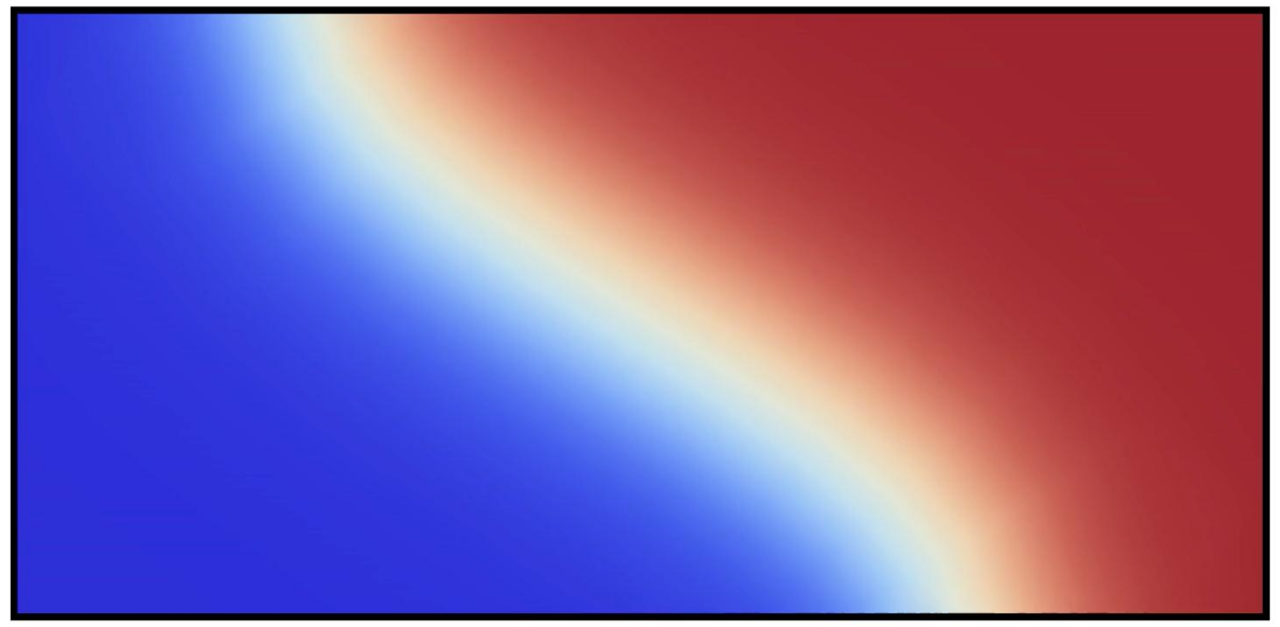
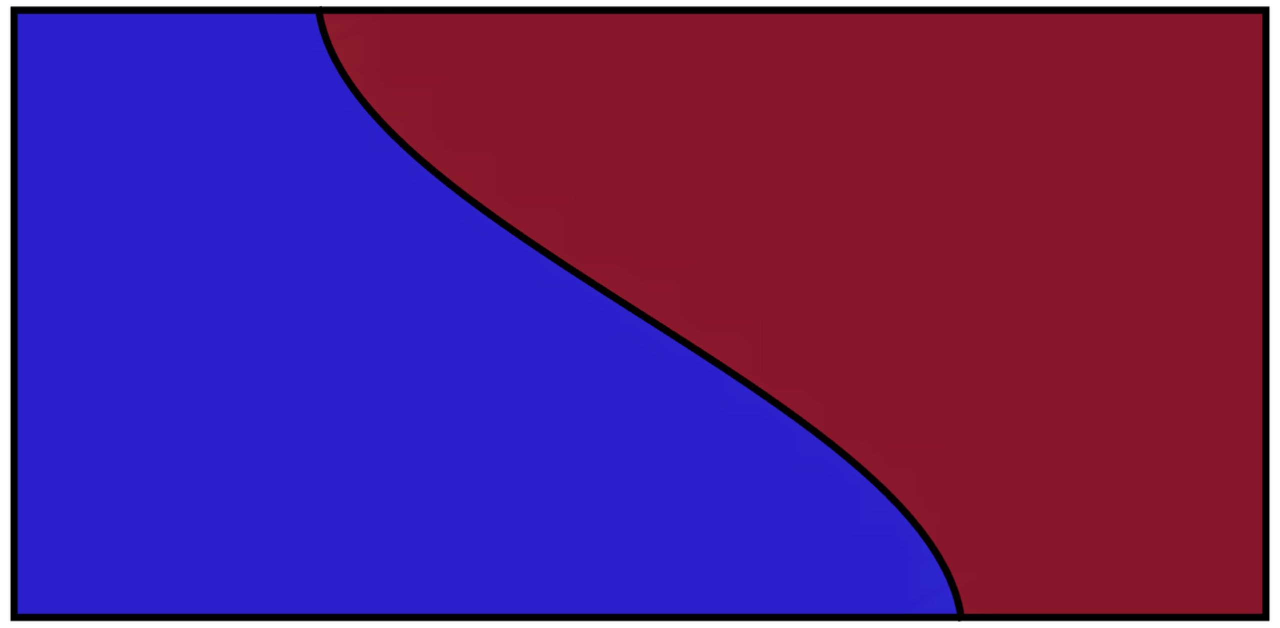
$\boldsymbol{n} \to \infty$



$\boldsymbol{\varepsilon} \to \mathbf{0}$

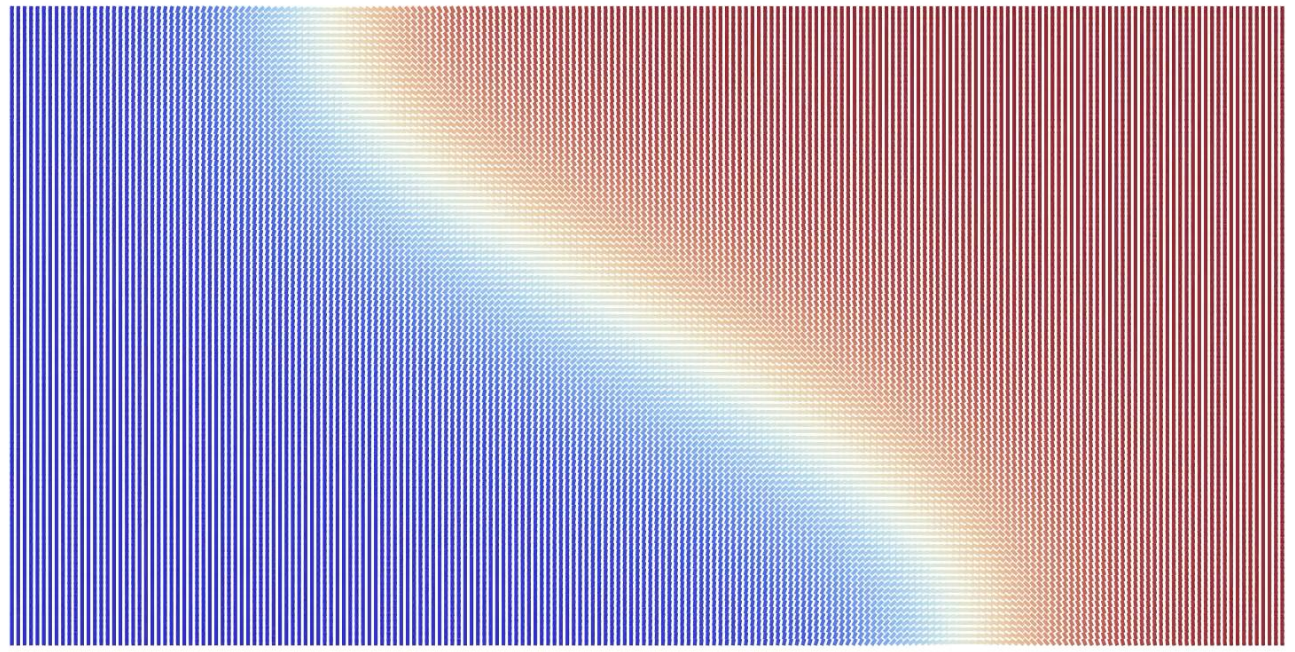
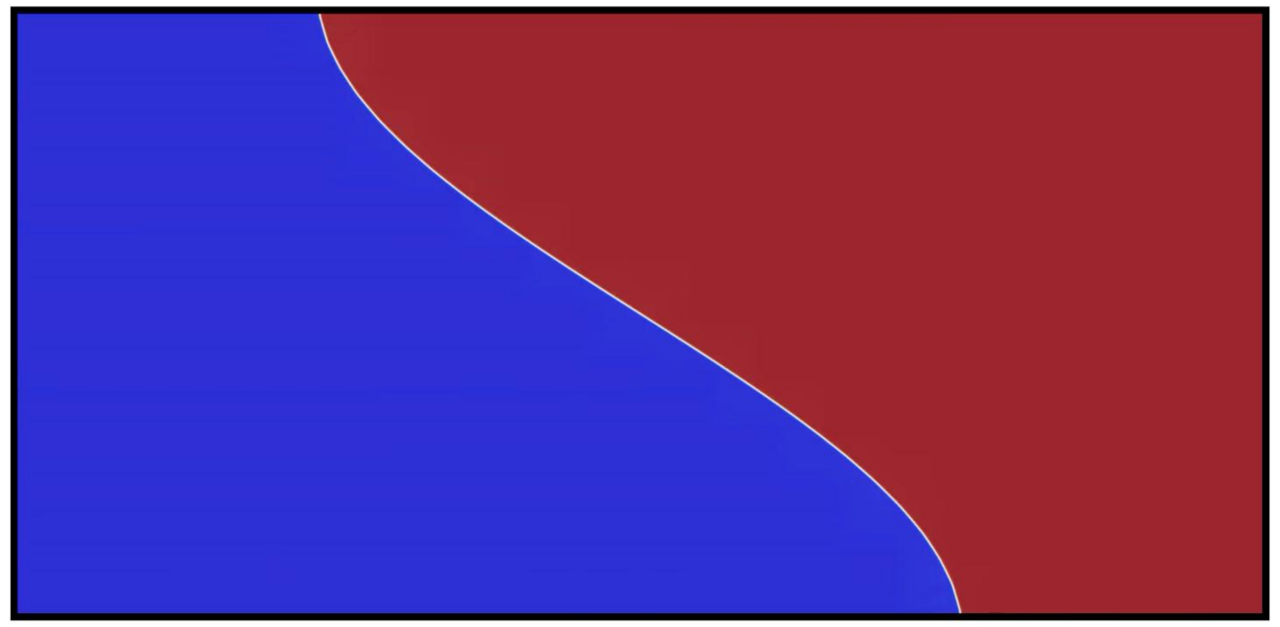
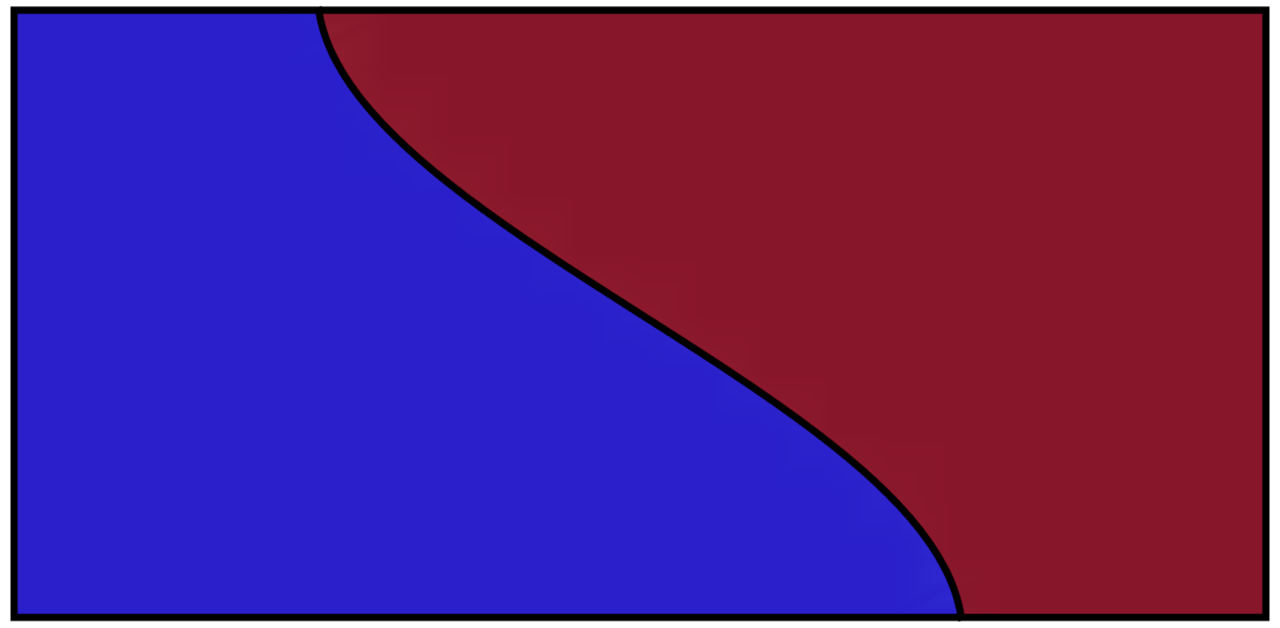
$$\boldsymbol{\varepsilon} \equiv \frac{micro\ length}{macro\ length}$$

The model hierarchy

Atomistic Models		Phase-field Models		Moving boundary Models
	$n \to \infty$		$\varepsilon \to 0$	

$$\varepsilon \equiv \frac{\textit{micro length}}{\textit{macro length}}$$

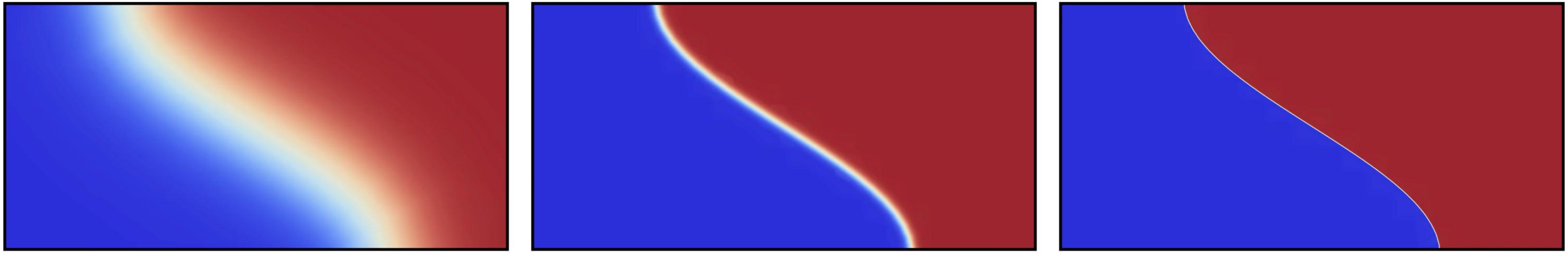
The model hierarchy

Atomistic Models		Phase-field Models		Moving boundary Models
	$n \to \infty$		$\varepsilon \to 0$	
Too many atoms!		$\varepsilon \equiv \dfrac{\textit{micro length}}{\textit{macro length}}$		Ignores microphysics!

Why phase-field?

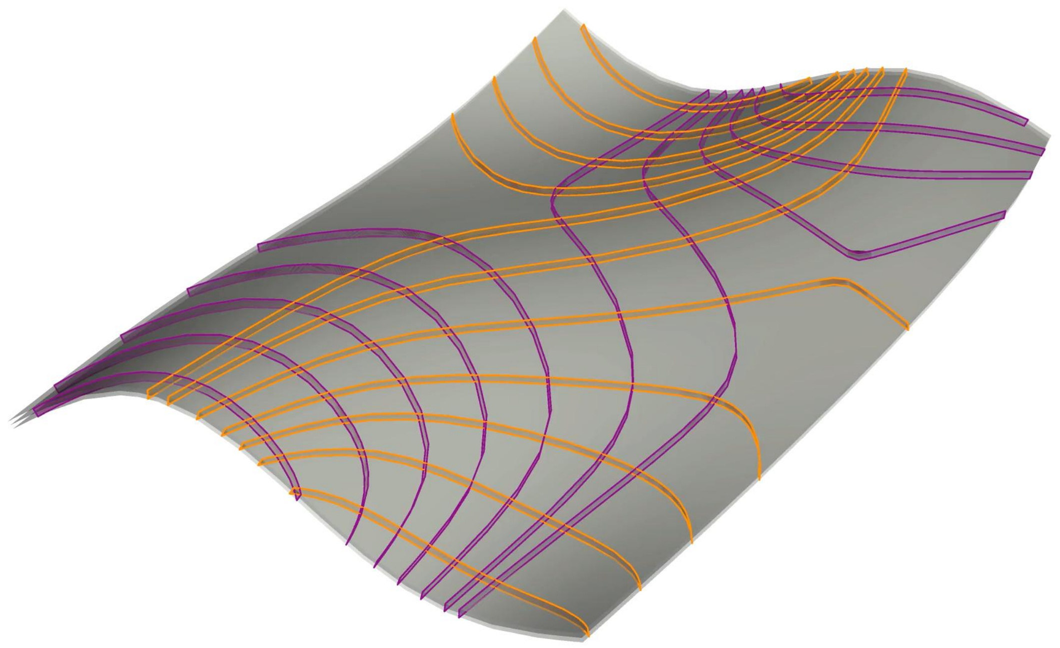
1. Physically motivated
2. Captures large *and* small scales
3. Easy to simulate

But multiscale! $\varepsilon \approx \frac{10^{-9} m}{10^{-1} m} = 10^{-8}$

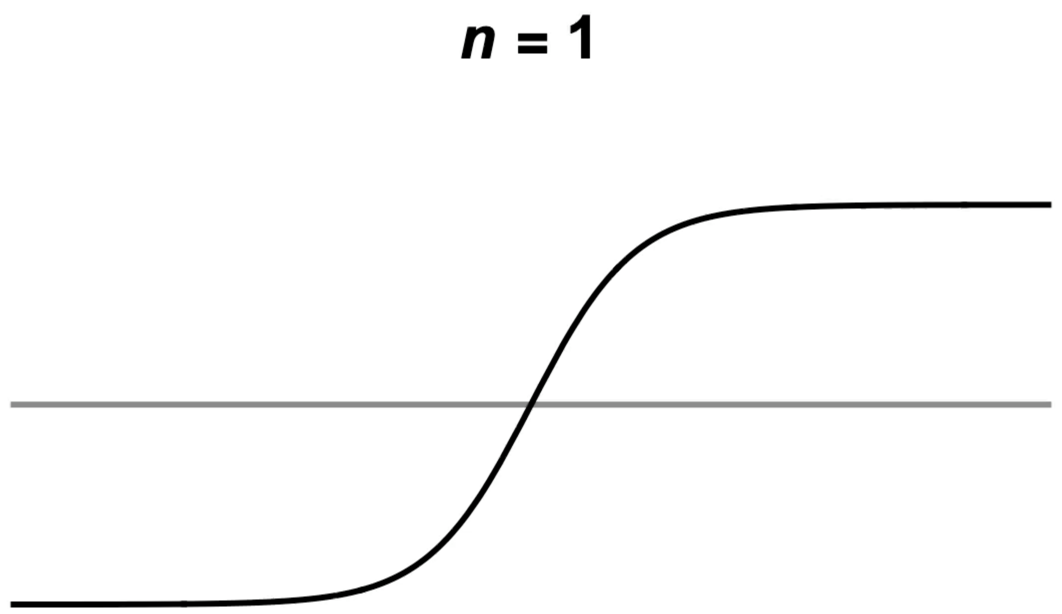


1. What's $\lim \varepsilon \to 0$?
2. What's the model *error* at finite ε?
3. What's the sim *error* and *cost* at finite ε, n?

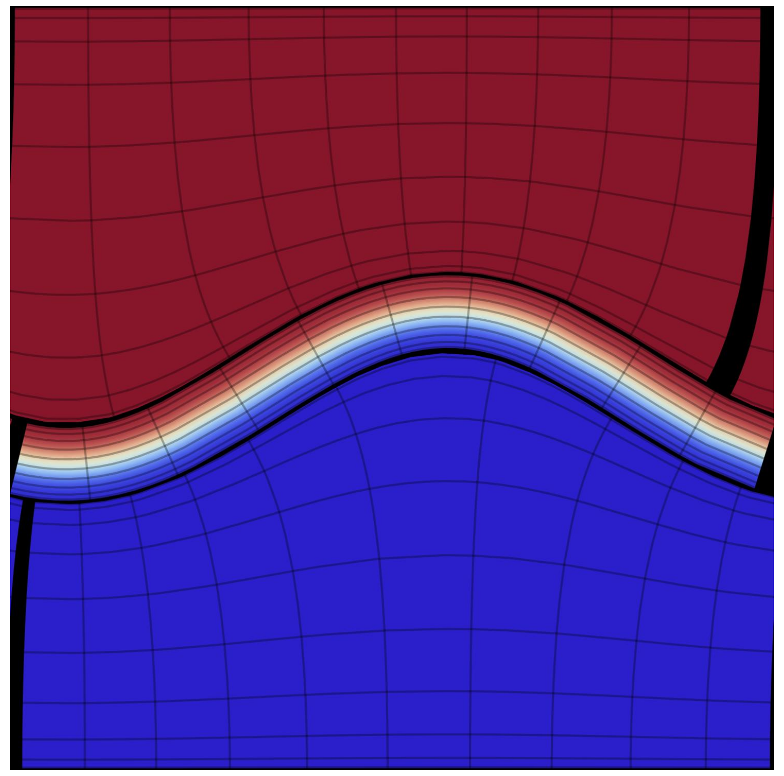
Three ideas to get $\varepsilon \to \mathbf{0}$



1. ***Differential geometry***
   to model boundaries as $\varepsilon \to \mathbf{0}$

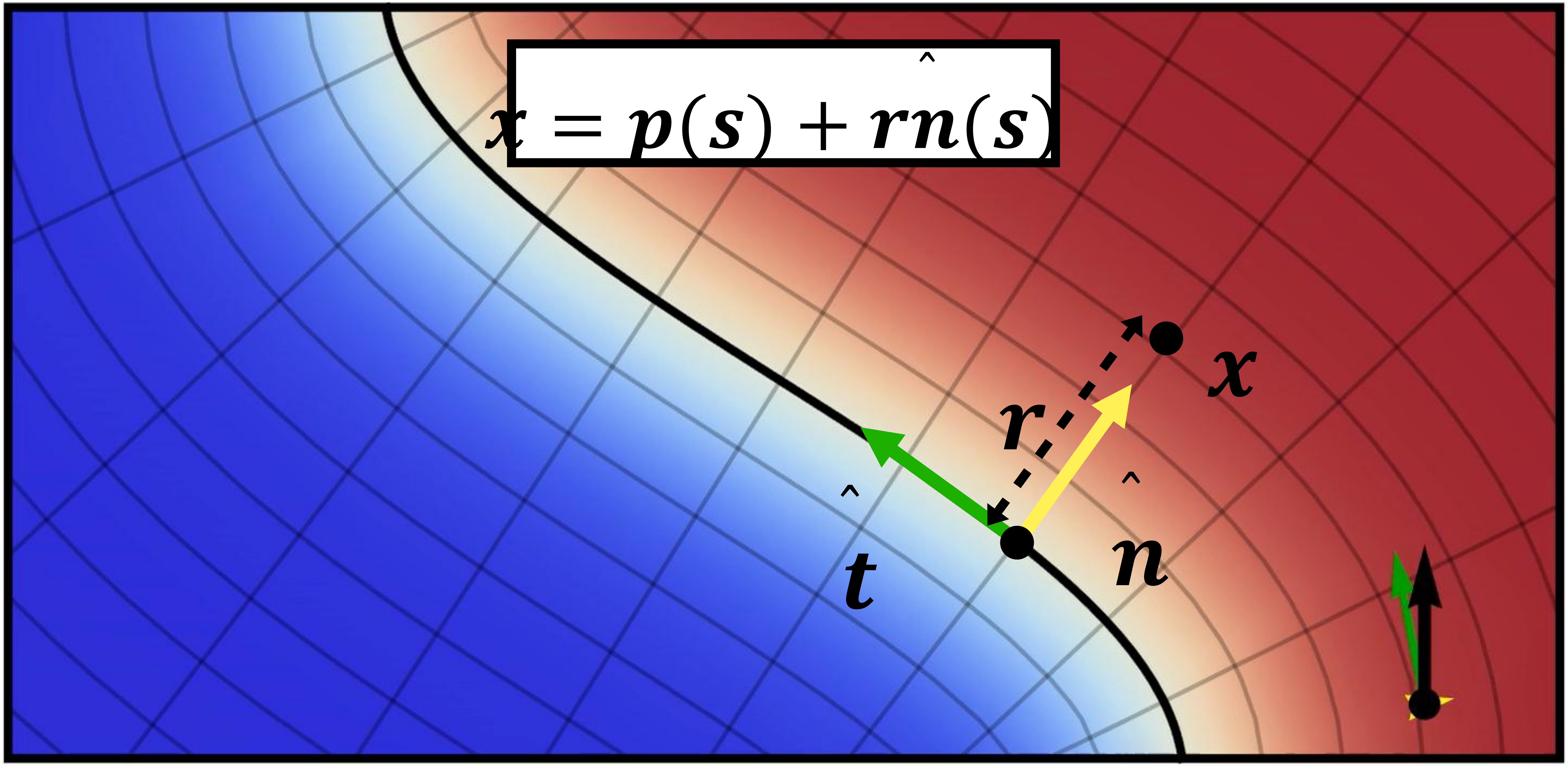


2. ***Spectral methods***
   to get $\mathcal{O}(n \log n)$ algorithms

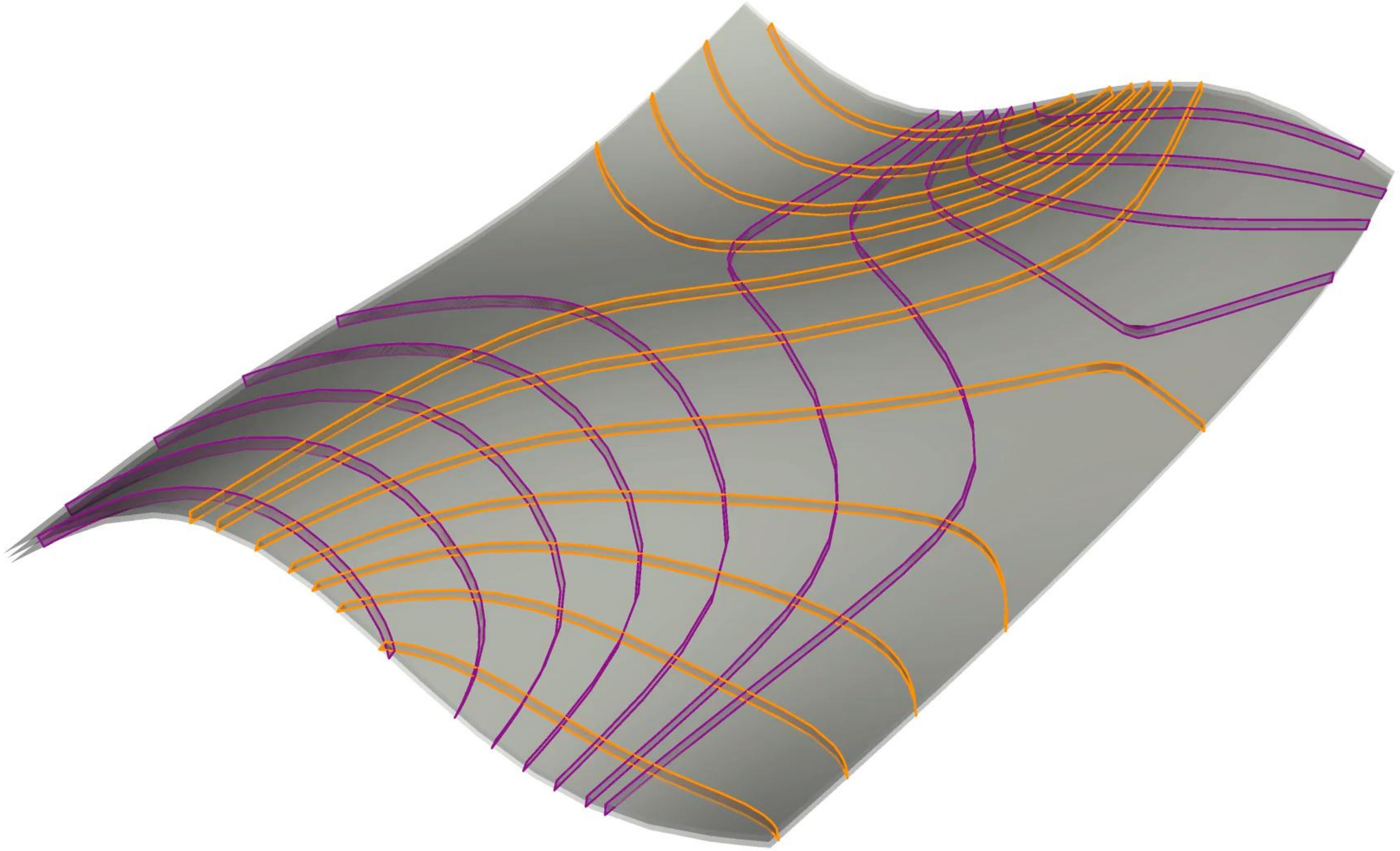


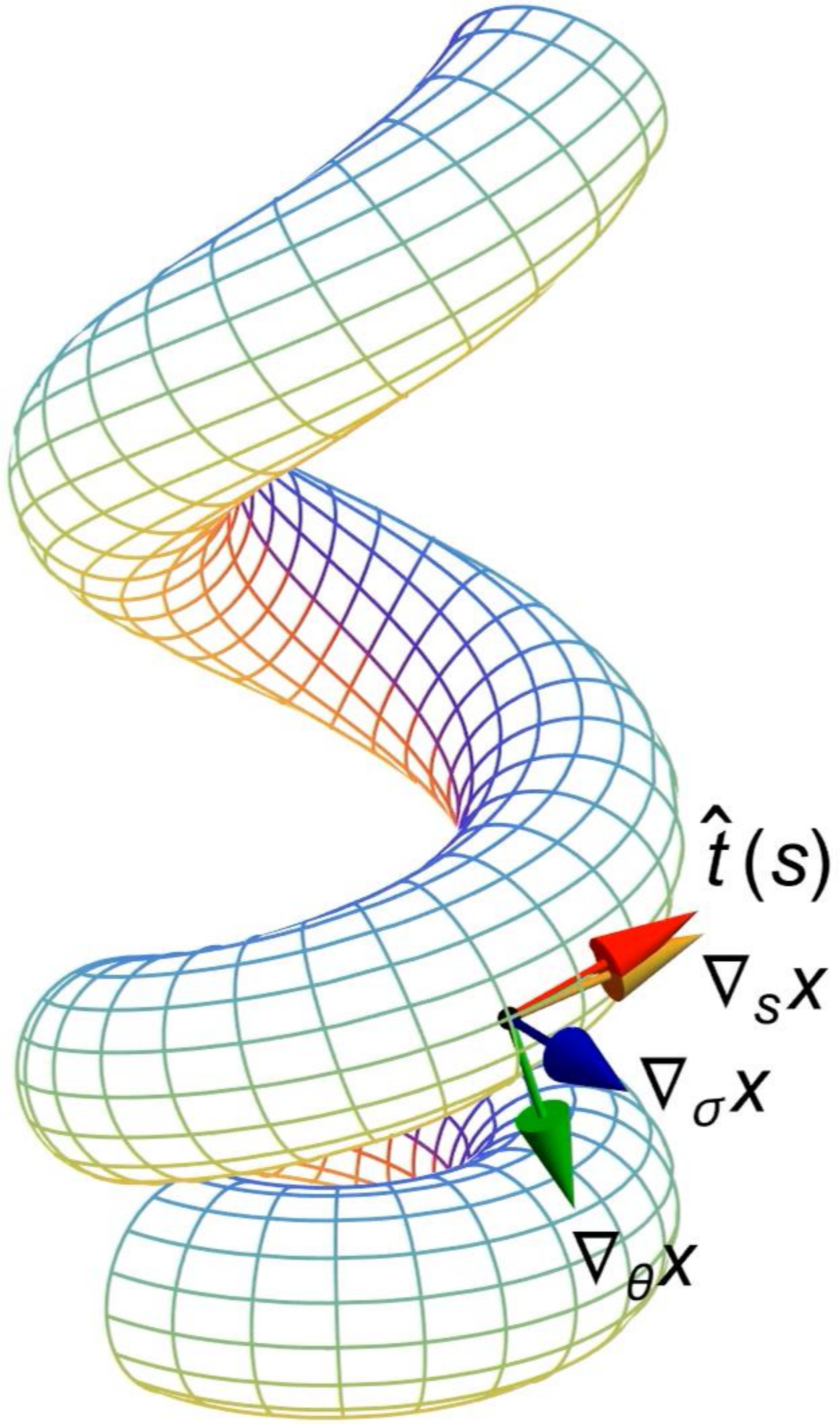
3. **Use *theory* to improve *algorithms***
   for optimal error and cost
   independent of ε

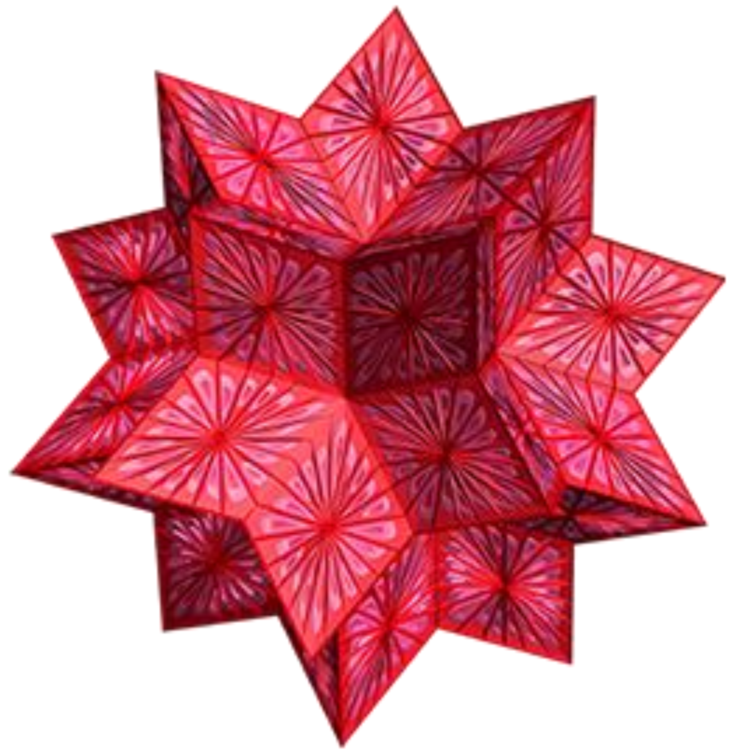
1. Signed Distance Coordinates near boundaries



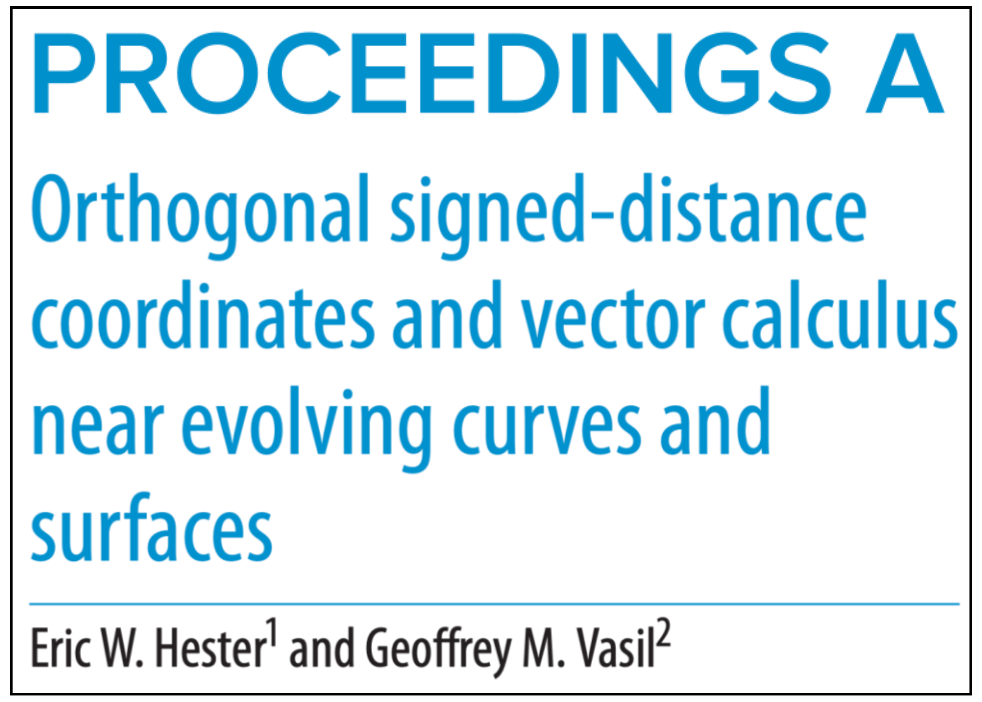
Automated Differential Geometry







$/ .\, \varepsilon \to 0$


PROCEEDINGS A

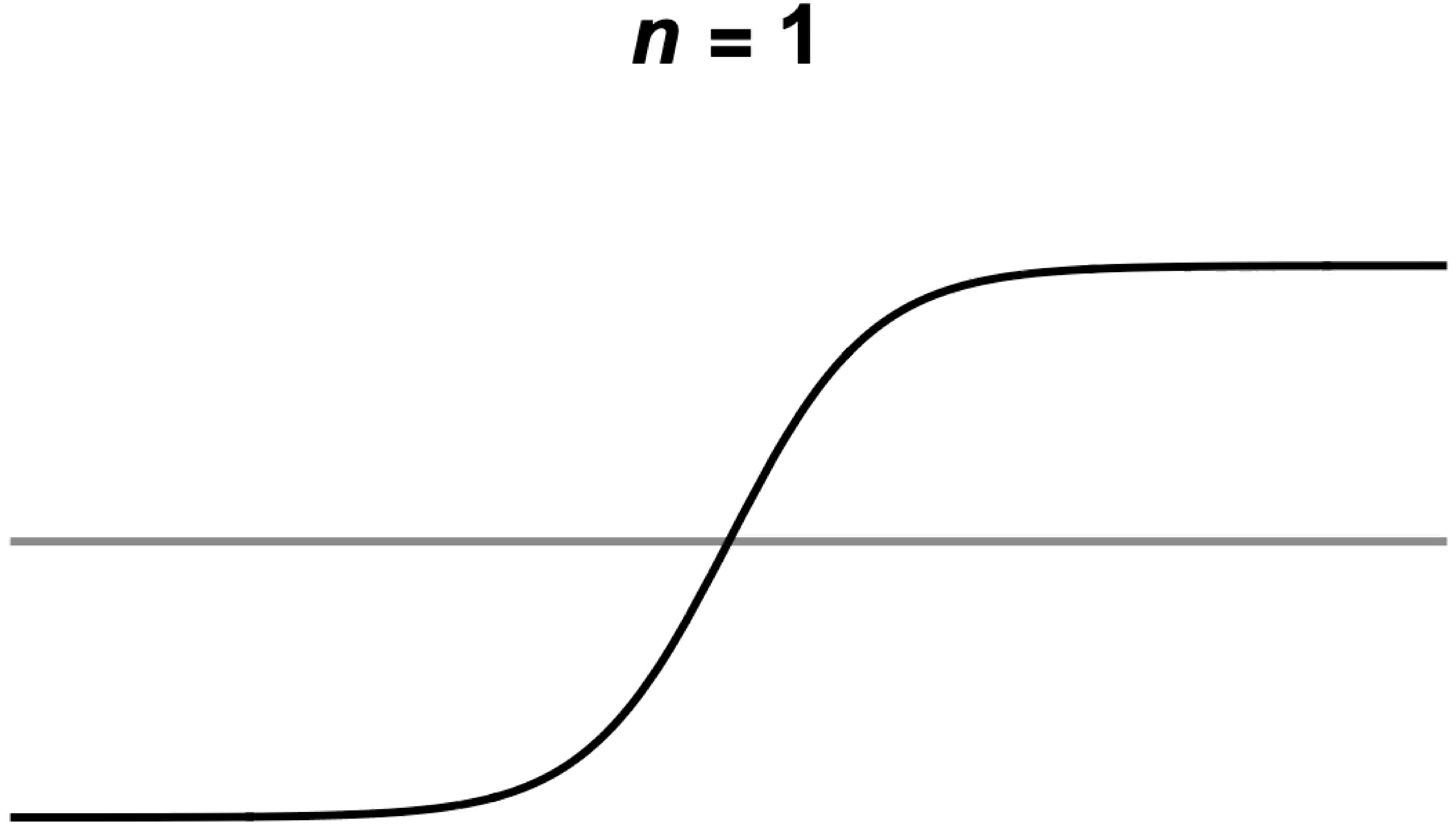
Orthogonal signed-distance coordinates and vector calculus near evolving curves and surfaces

Eric W. Hester[1] and Geoffrey M. Vasil[2]

Signed Distance Coords $p(s) + r\hat{n}(s)$
for any dimension

Automate $\varepsilon \to 0$
for any physics!

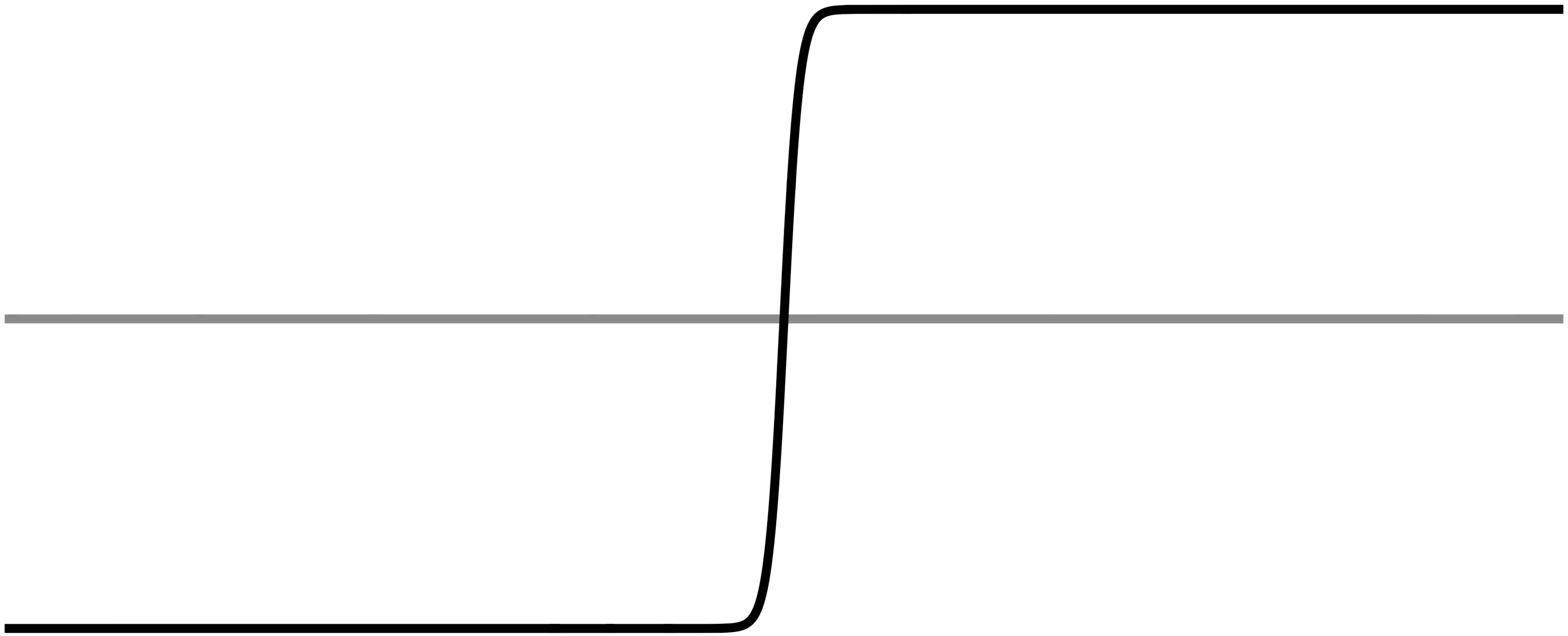
2. Algorithms: Spectral methods
Approximate with orthogonal polynomials



Error $\sim e^{-an} \ll n^{-b}$ **Cost** $\sim n\log n$

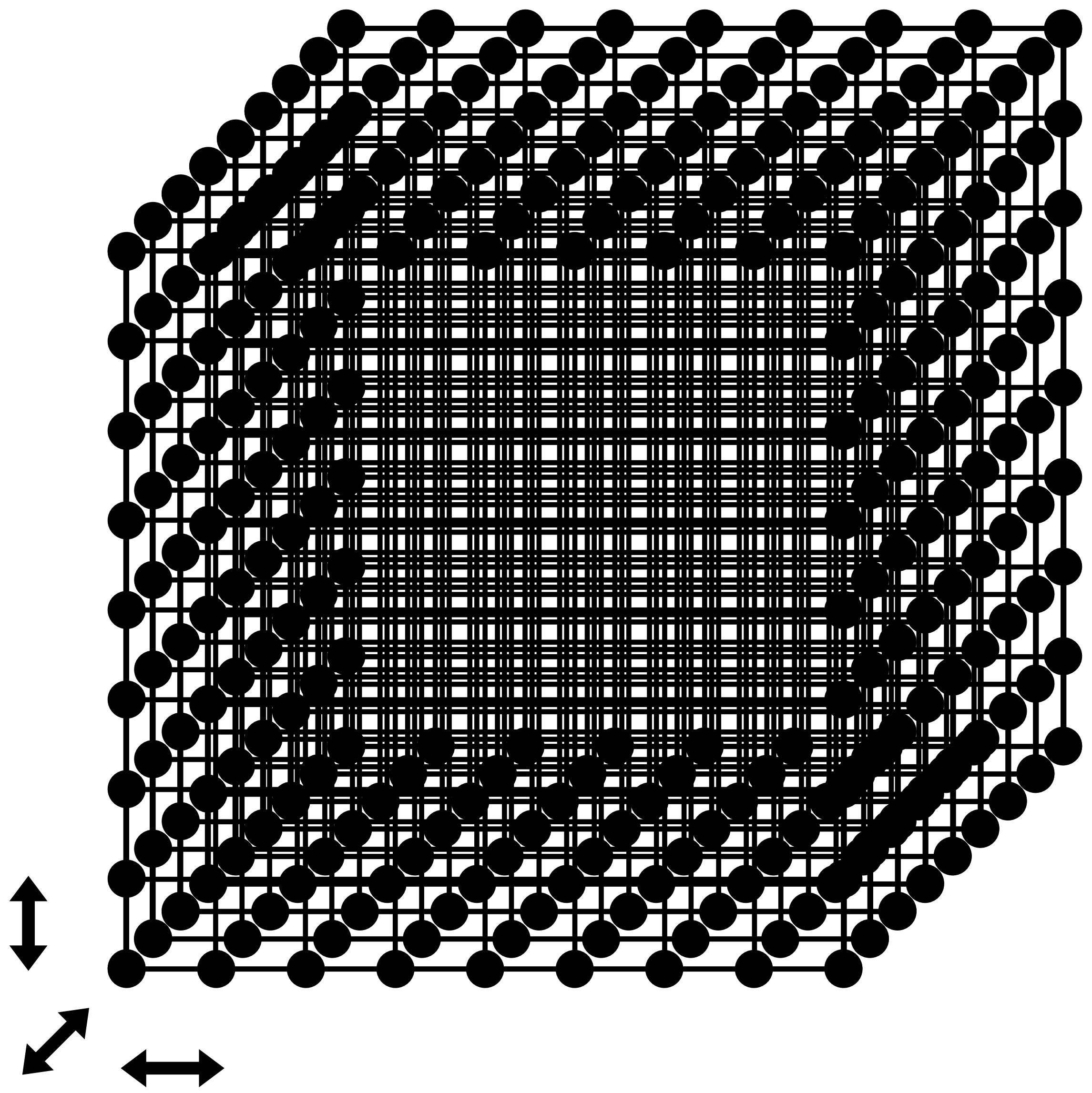
But Gibbs means *naively* $n \sim \varepsilon^{-1}$!

$\varepsilon = 10^{-2}$ *n* = 1



Uniform resolution bad for $\varepsilon \to 0$!

3D Cost: $\Delta x^{-3} \Delta t^{-1}$

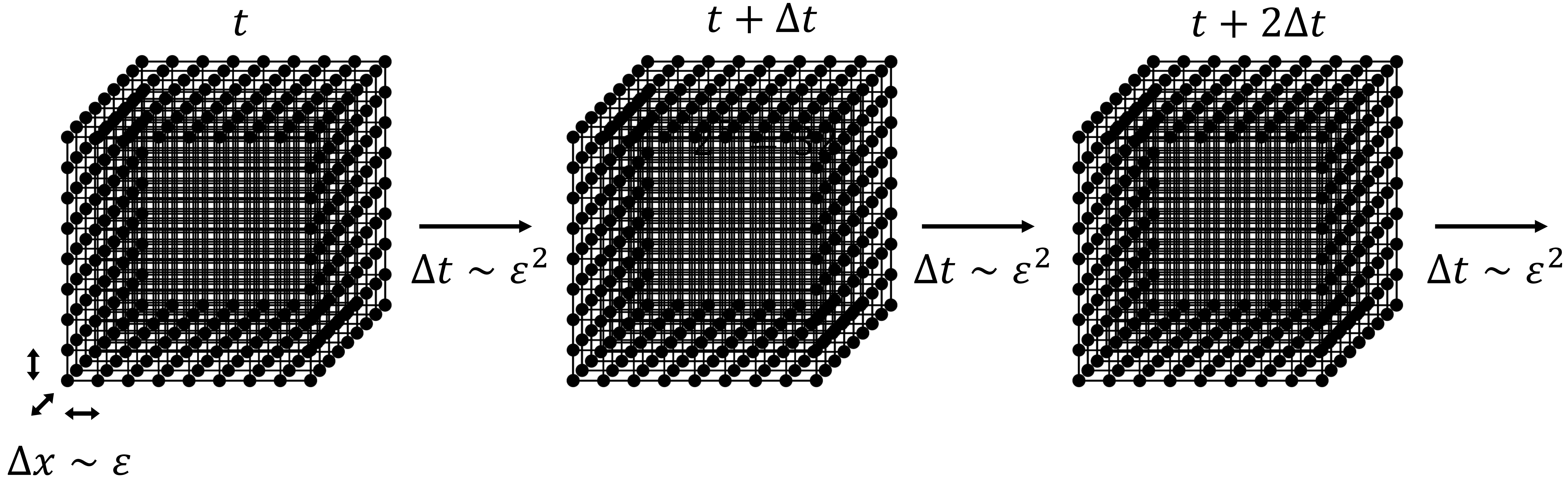


$\Delta x \sim \varepsilon$

Uniform resolution bad for $\varepsilon \to 0$!

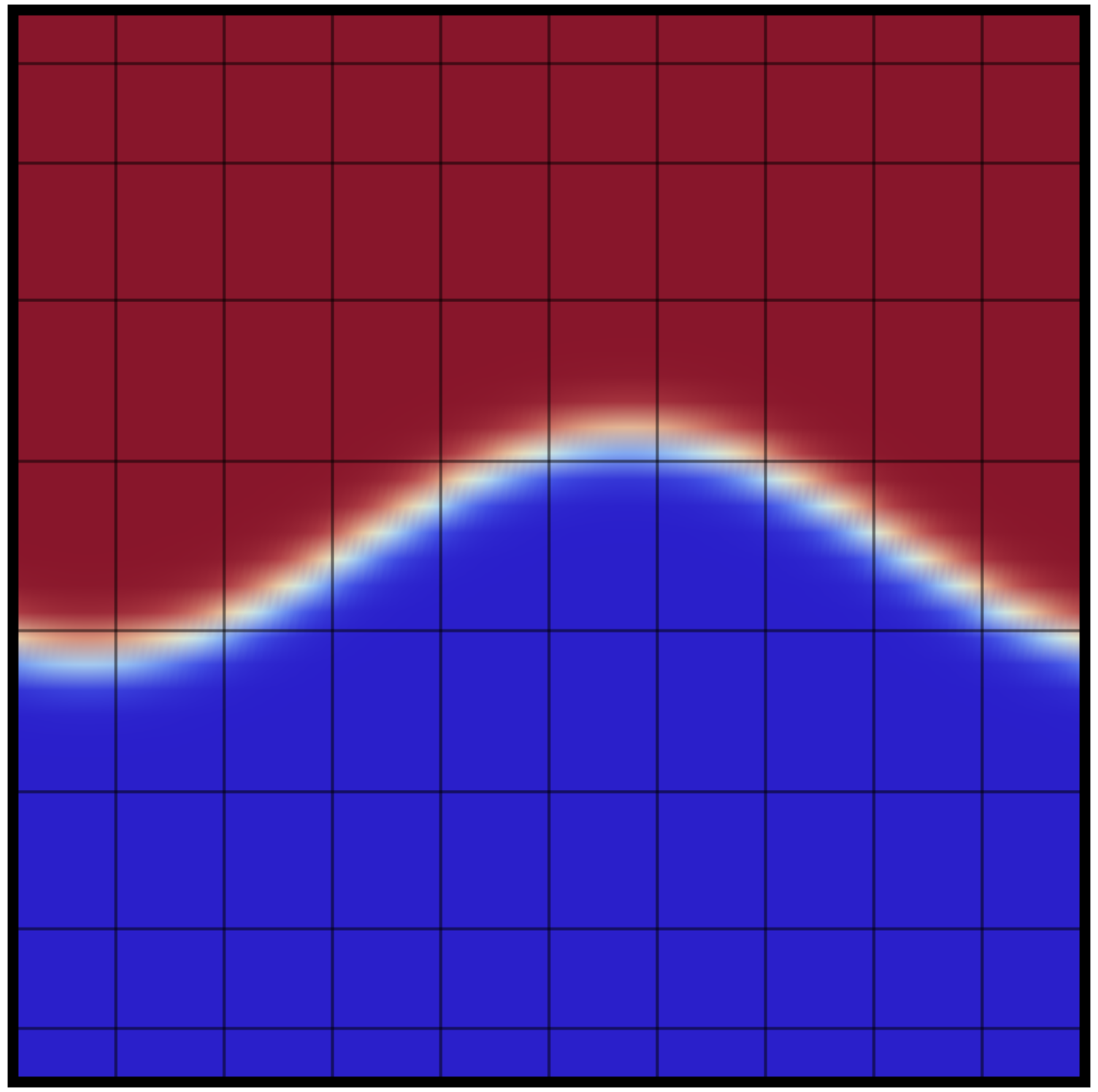
3D Cost: $\Delta x^{-3} \Delta t^{-1} \sim \varepsilon^{-5}$!

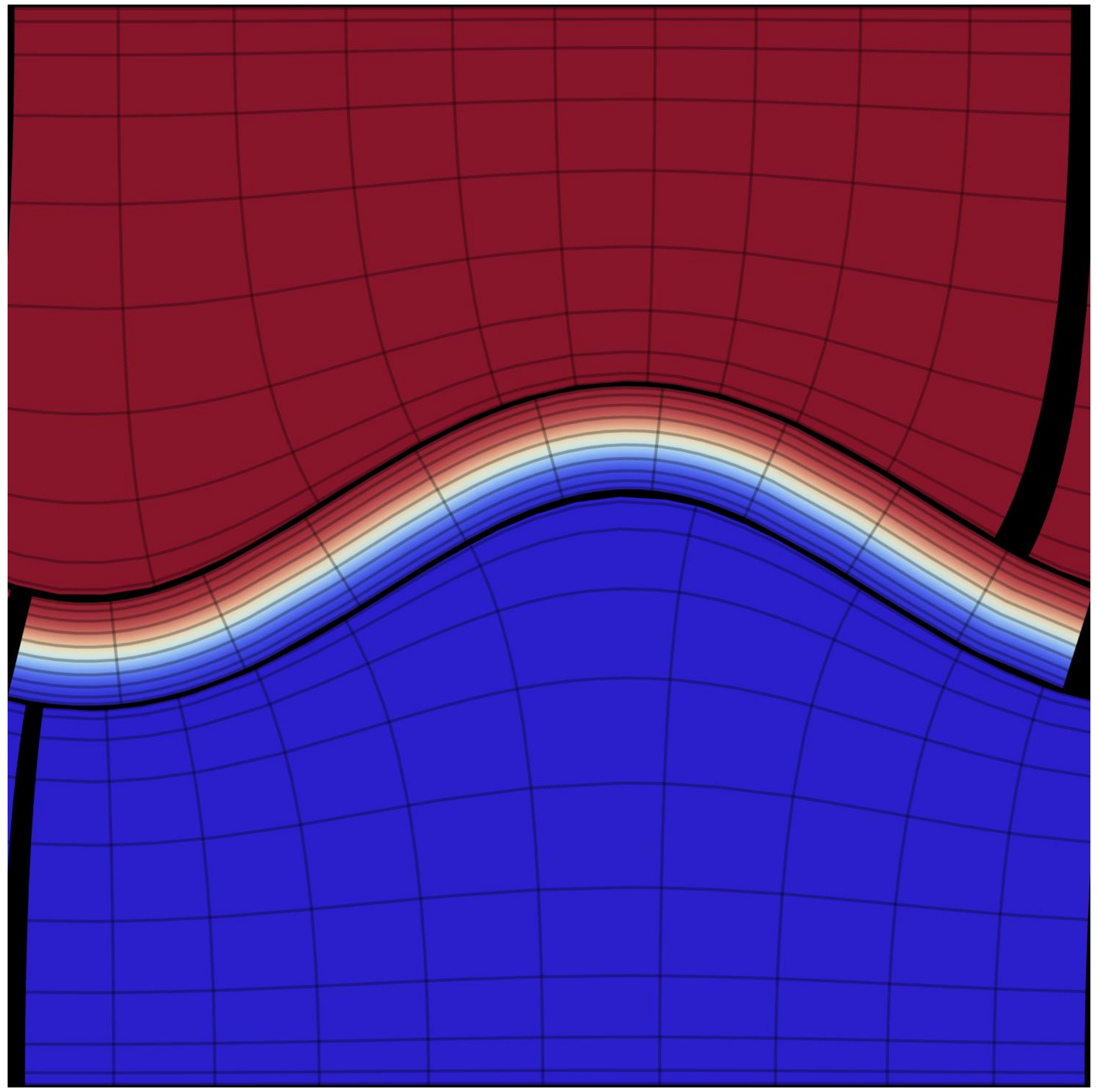
$$\mathit{Error} \times \frac{1}{2} \Rightarrow \mathit{Cost} \times 2^5$$



3. Towards Optimal Complexity

Use the theory to design better algorithms!



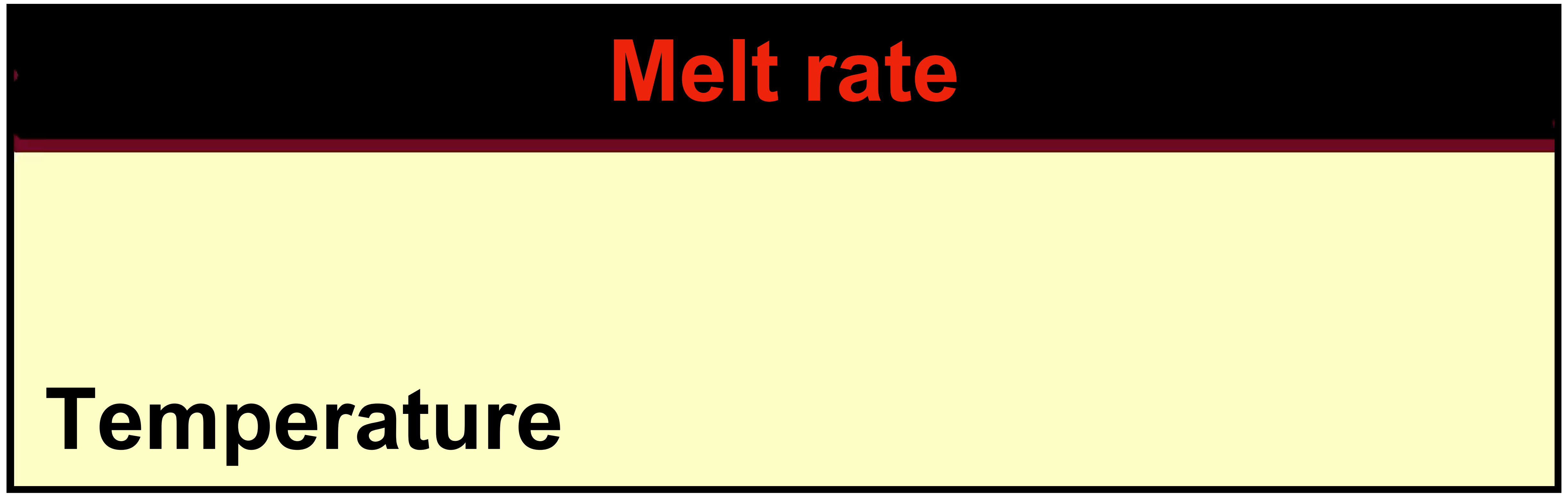


1. Asymptotics to accelerate $\lim \varepsilon \to 0$
2. Coordinate transforms improve resolution Δx
3. Optimal preconditioning to improve Δt

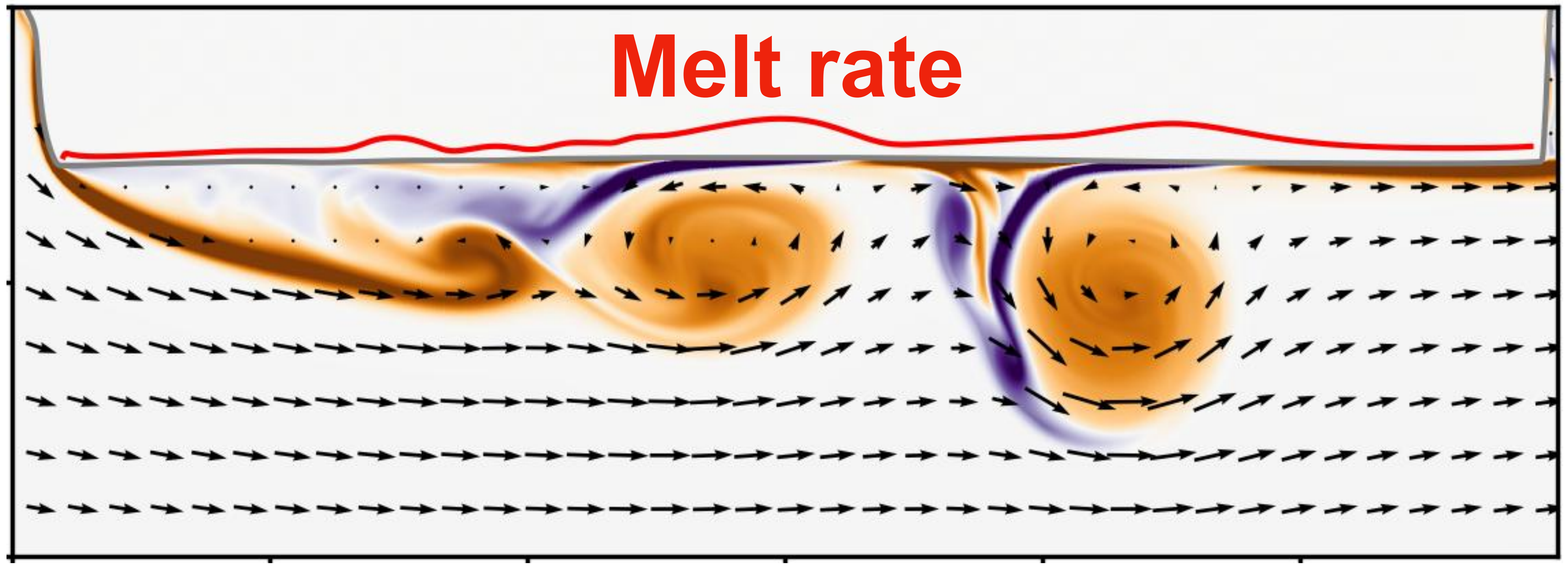
And what does this tell us about icebergs?

Simulations reproduce melt rates

Melt rate



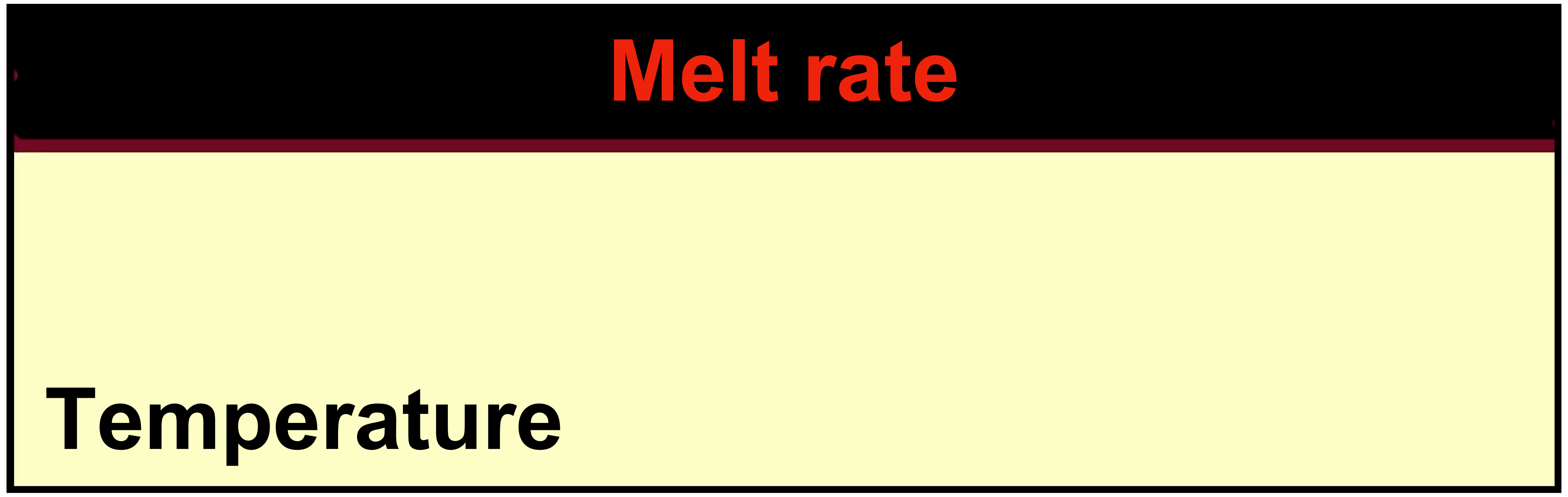
Time

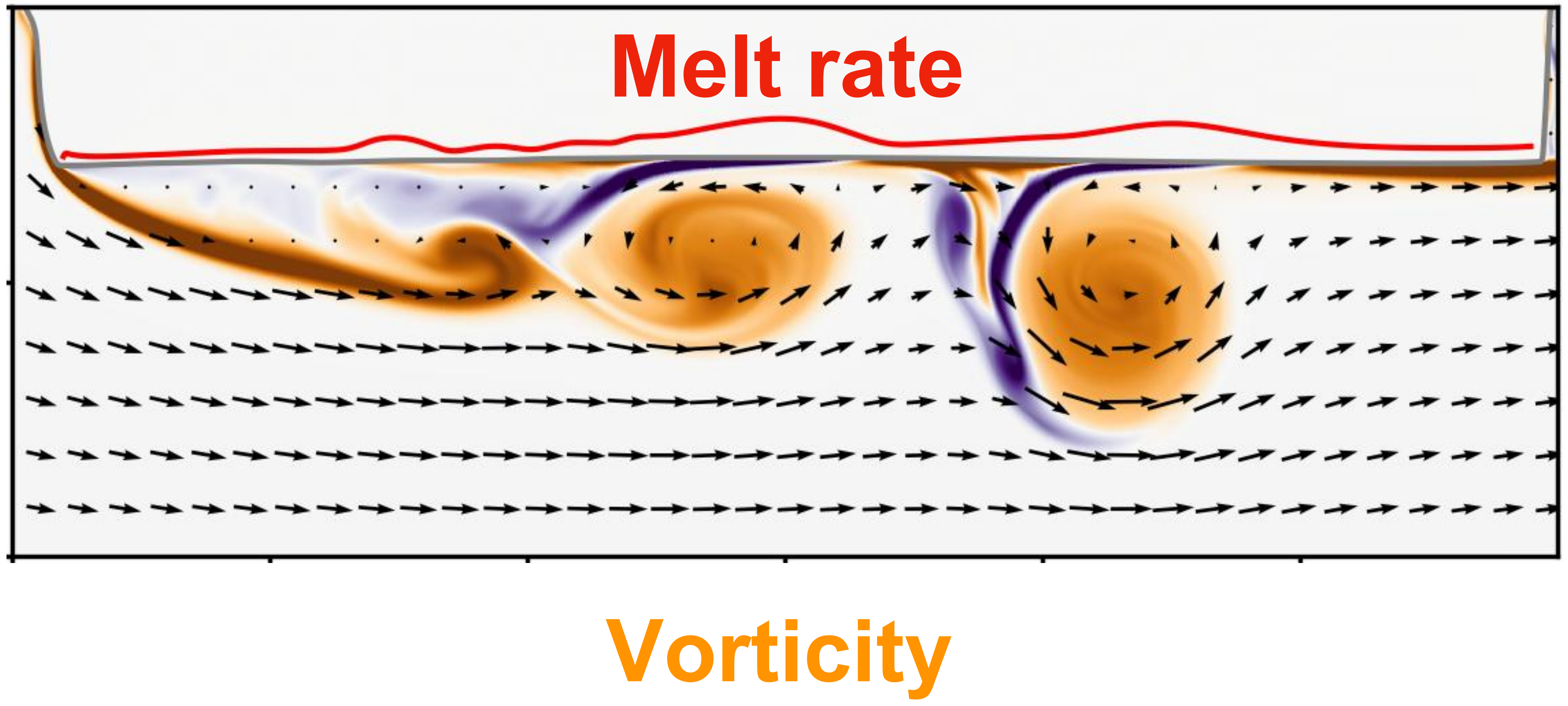


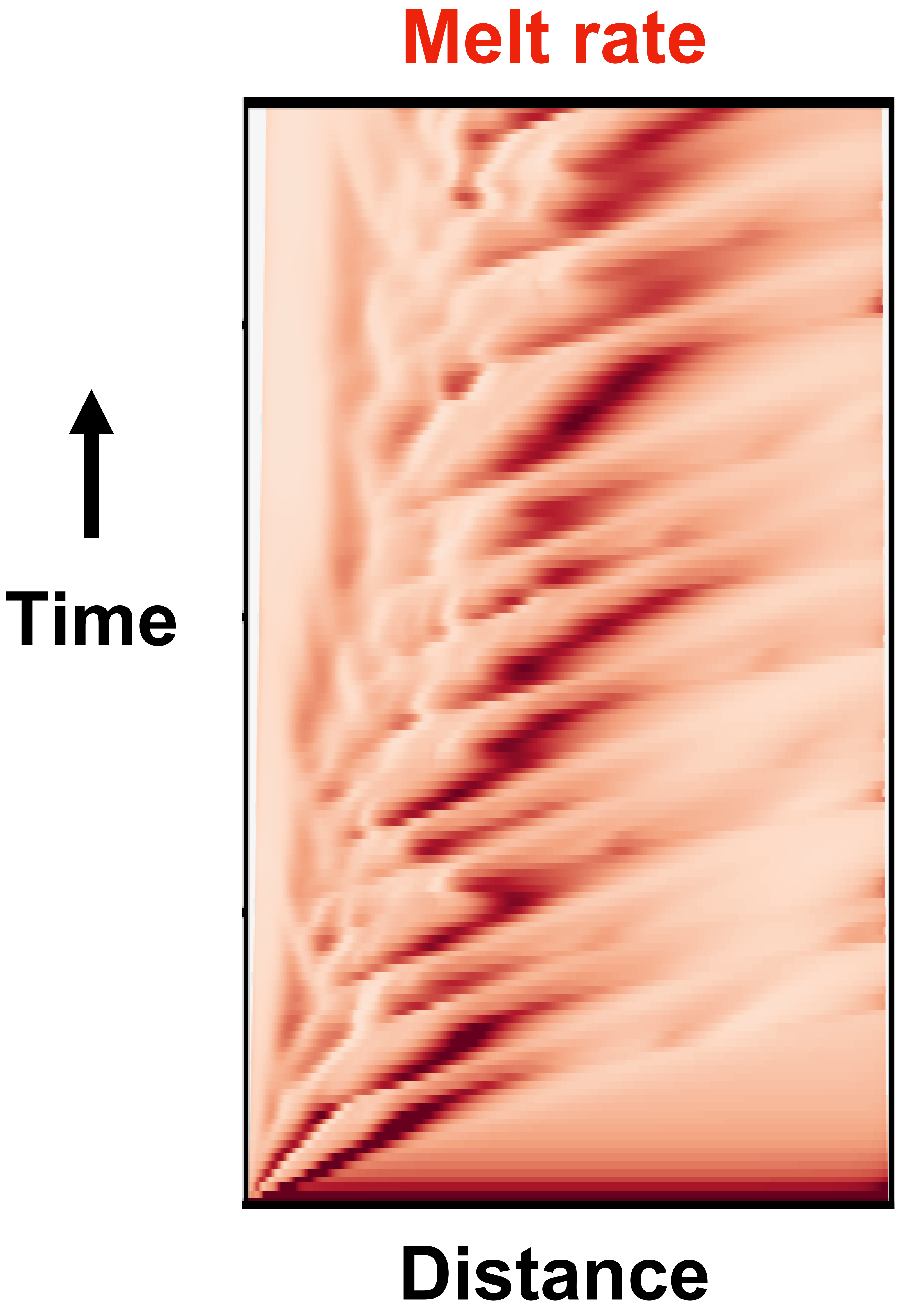
Vorticity

Distance

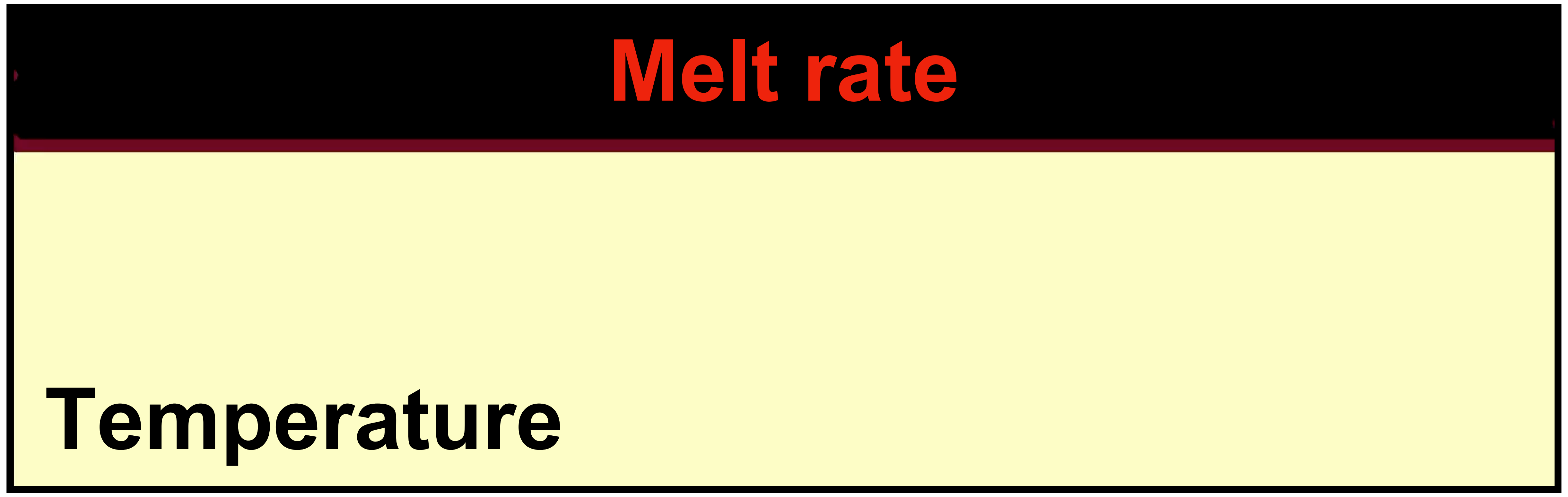
Simulations reproduce melt rates

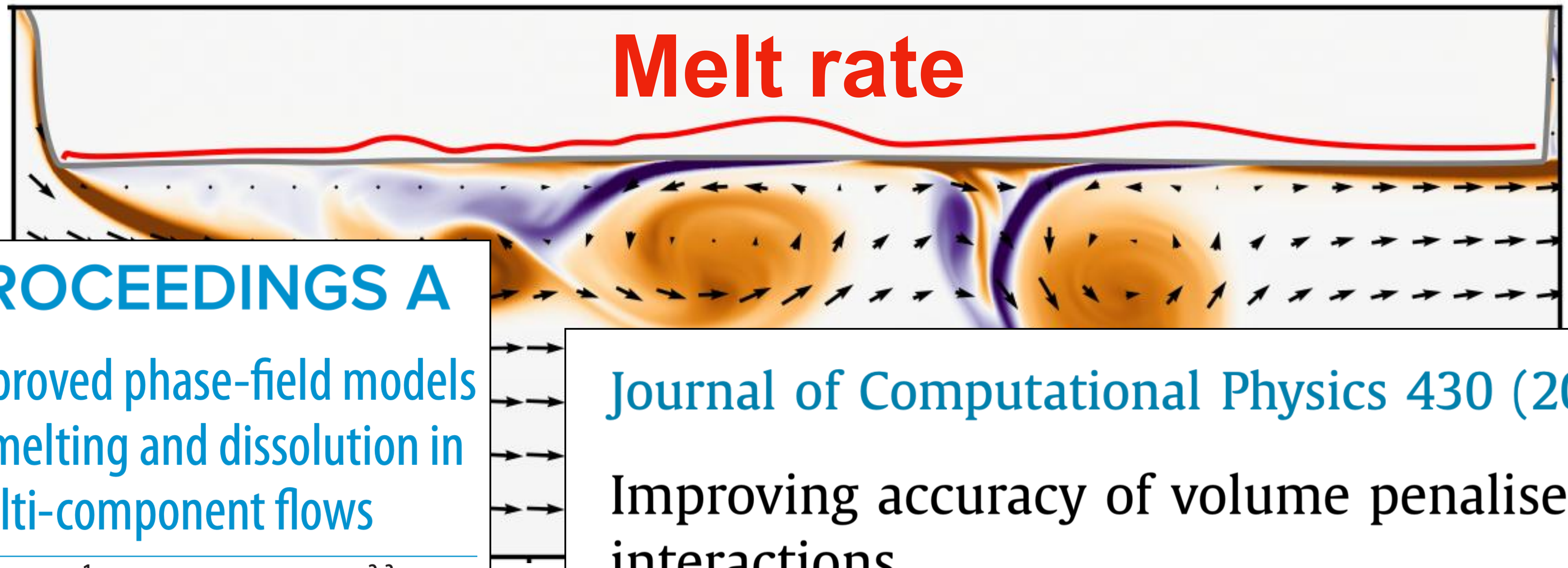


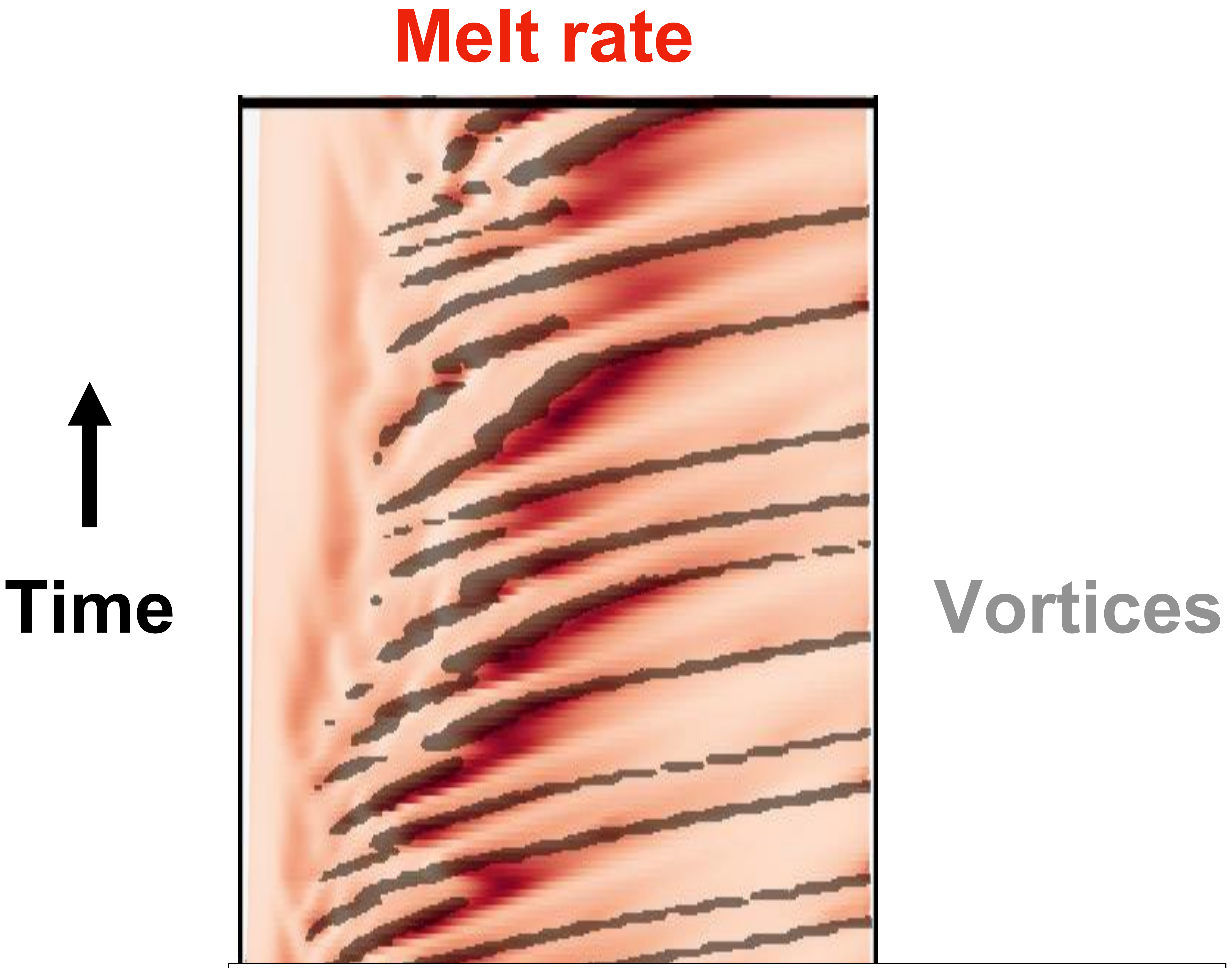




Simulations reproduce melt rates







PROCEEDINGS A

Improved phase-field models of melting and dissolution in multi-component flows

Eric W. Hester[1], Louis-Alexandre Couston[2,3], Benjamin Favier[4], Keaton J. Burns[5,6] and Geoffrey M. Vasil[1]

Journal of Computational Physics 430 (2021) 110043

Improving accuracy of volume penalised fluid-solid interactions

Eric W. Hester[a,*], Geoffrey M. Vasil[a], Keaton J. Burns[b,c]

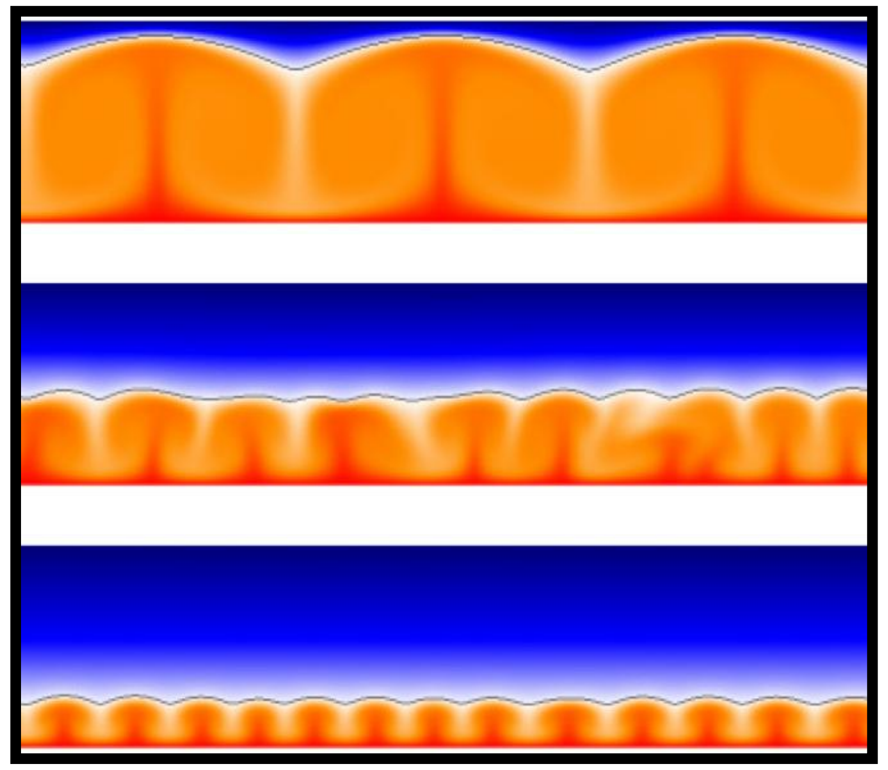
PHYSICAL REVIEW FLUIDS **6**, 023802 (2021)

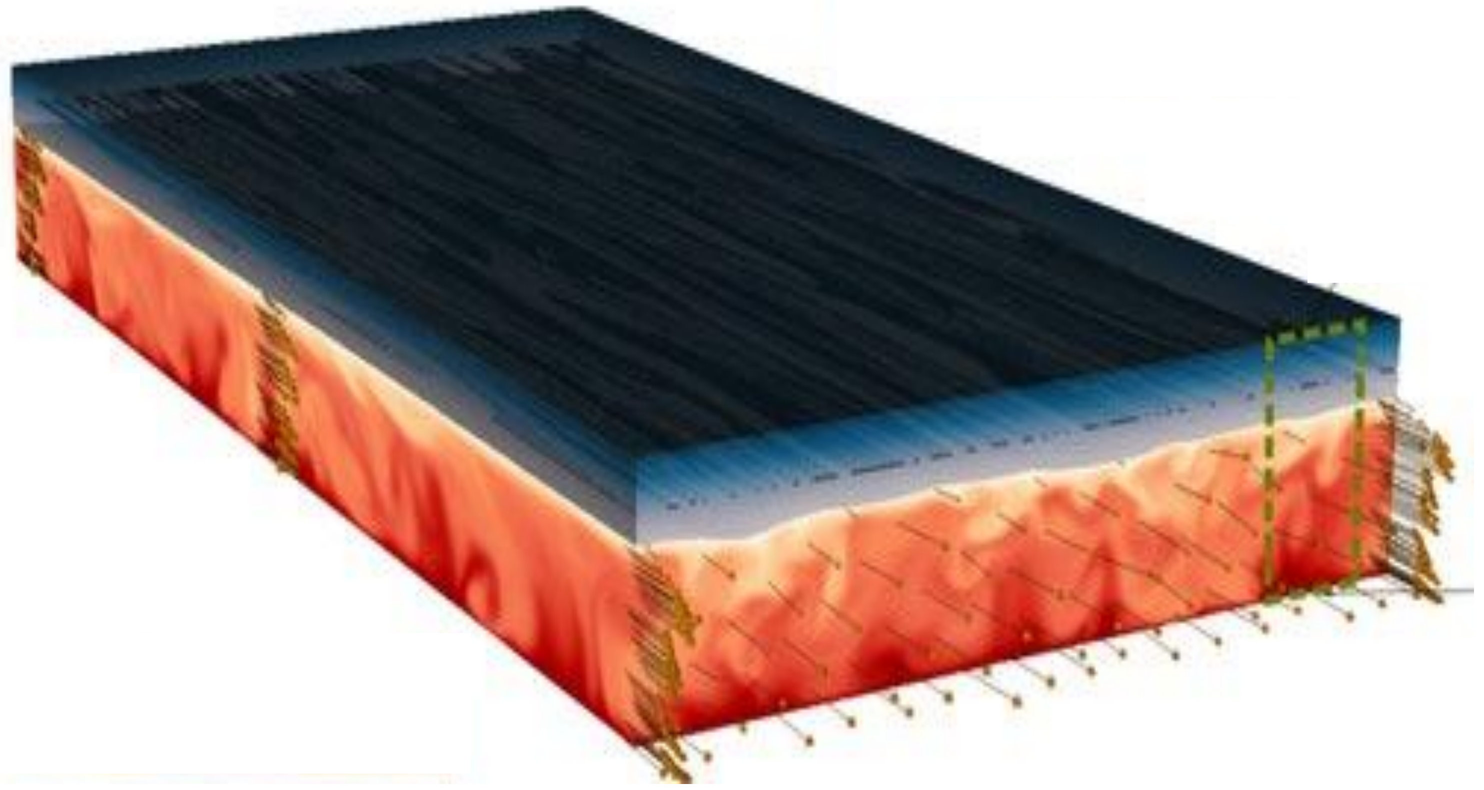
Aspect ratio affects iceberg melting

Eric W. Hester*, Craig D. McConnochie, Claudia Cenedese, Louis-Alexandre Couston, Geoffrey Vasil

Editors' Suggestion | Featured in Physics

More Ice-ocean interactions!





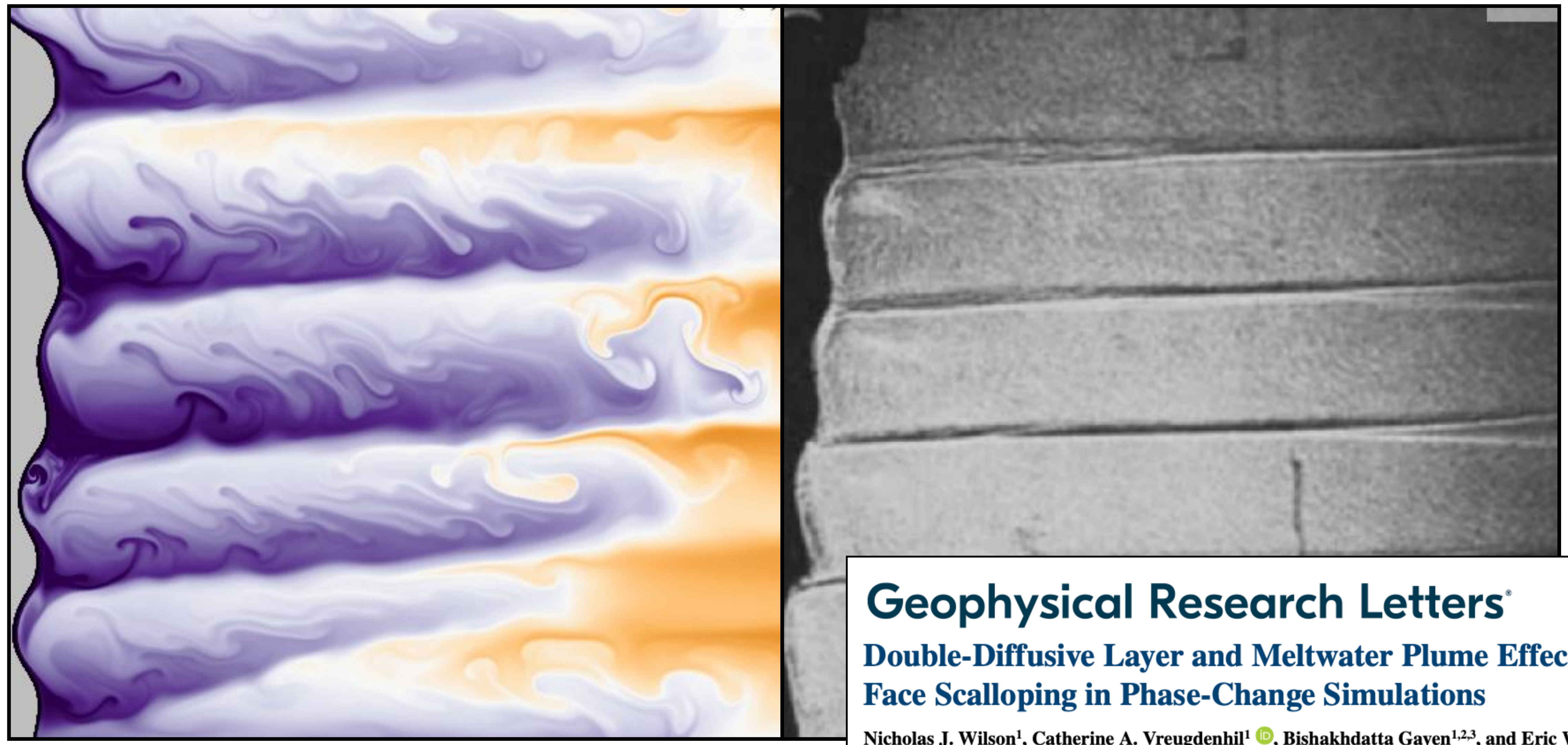
PHYSICAL REVIEW FLUIDS **5**, 023501 (2020)

Bistability in Rayleigh-Bénard convection with a melting boundary

J. Purseed, B. Favier,* L. Duchemin E. W. Hester

Topography generation by melting and freezing in a turbulent shear flow

Louis-Alexandre Couston[1,2,3,†], Eric Hester[4], Benjamin Favier[5], John R. Taylor[2], Paul R. Holland[1] and Adrian Jenkins[1,6]



Geophysical Research Letters

Double-Diffusive Layer and Meltwater Plume Effects on Ice Face Scalloping in Phase-Change Simulations

Nicholas J. Wilson[1], Catherine A. Vreugdenhil[1], Bishakhdatta Gayen[1,2,3], and Eric W. Hester[4]

Can we reproduce roughness?

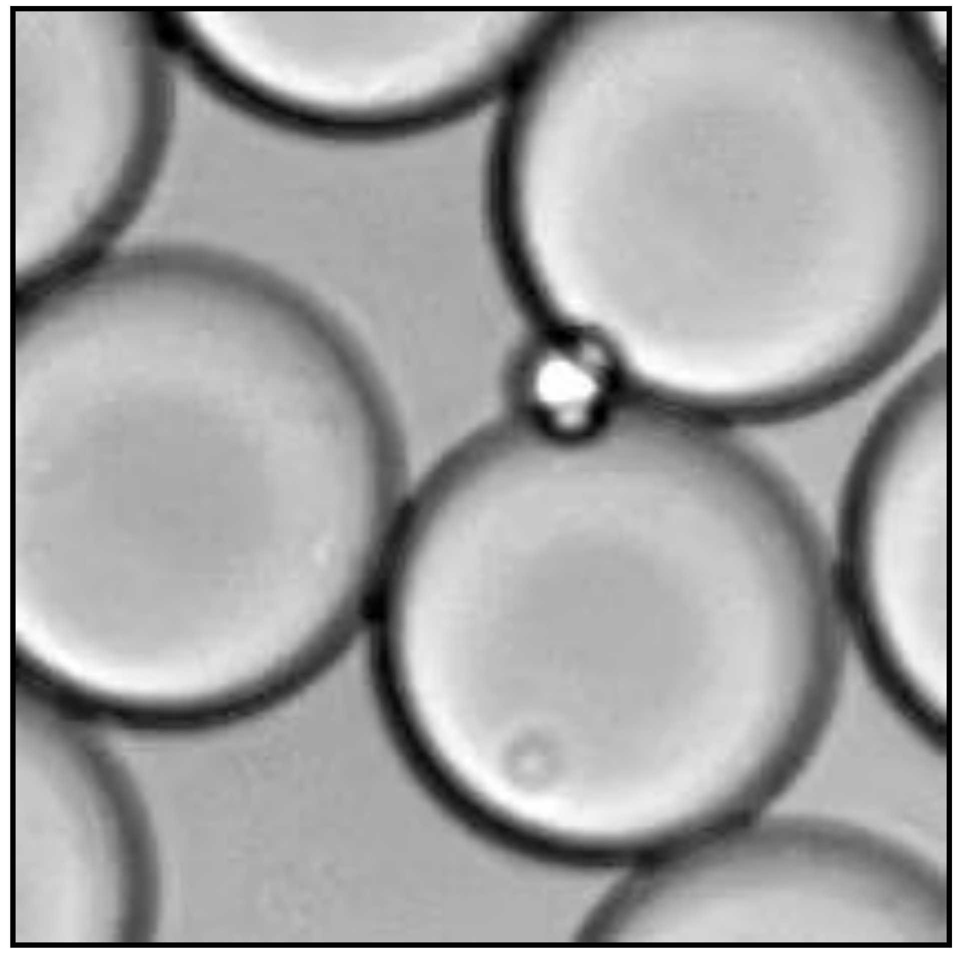
Moving, rolling, calving…?

Accurate models of sea ice?

More multiphase modelling!

Dead Water: Boat Drag via Internal Waves

Nanovial Manufacture via Temperature-induced Phase Separation



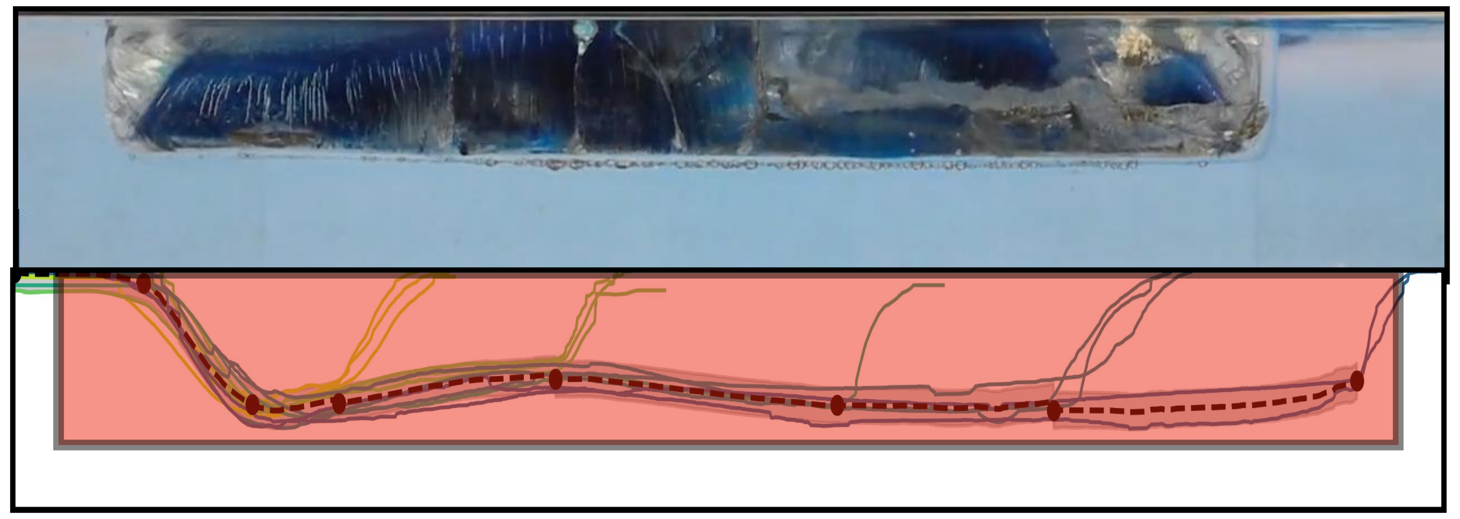
PNAS RESEARCH ARTICLE | APPLIED MATHEMATICS

Fluid dynamics alters liquid–liquid phase separation in confined aqueous two-phase systems

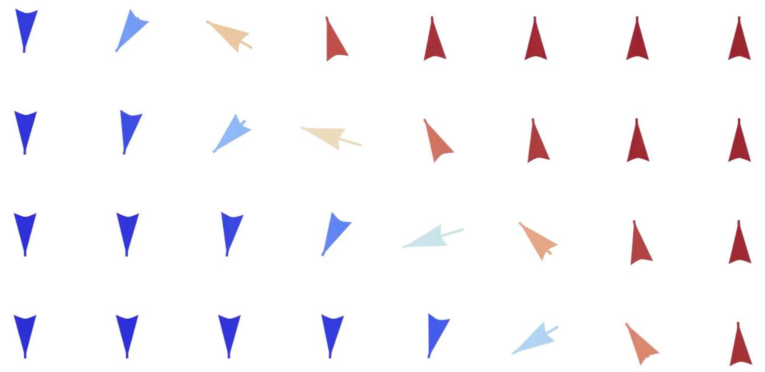
Eric Hester[a,b], Sean Carney[a,b], Vishwesh Shah[c], Alyssa Arnheim[c], Bena Patel[c], Dino Di Carlo[b,c,d], and Andrea L. Bertozzi[a,b,d,1]

Modelling Multiscale Multiphase Matter

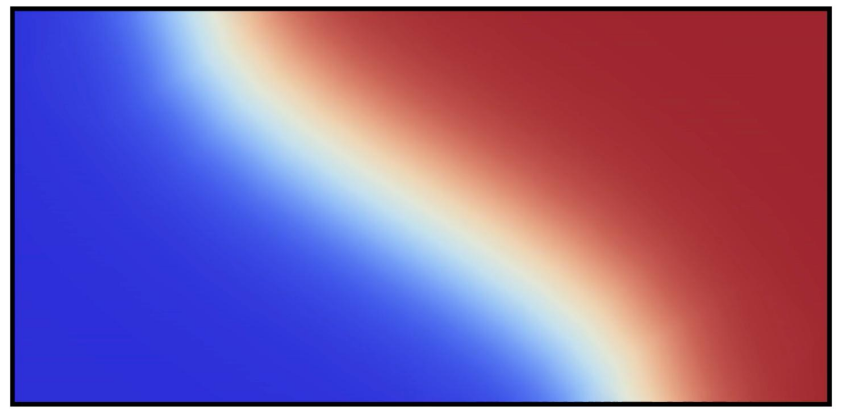
Can we predict iceberg melt?



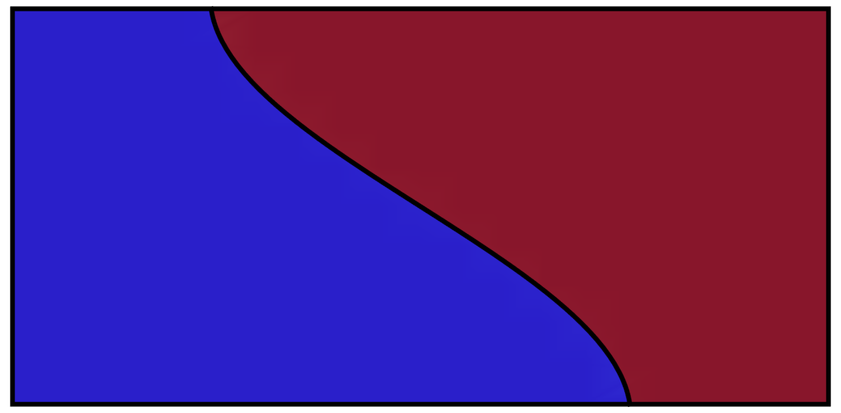
How to build a better model?



Atomistic

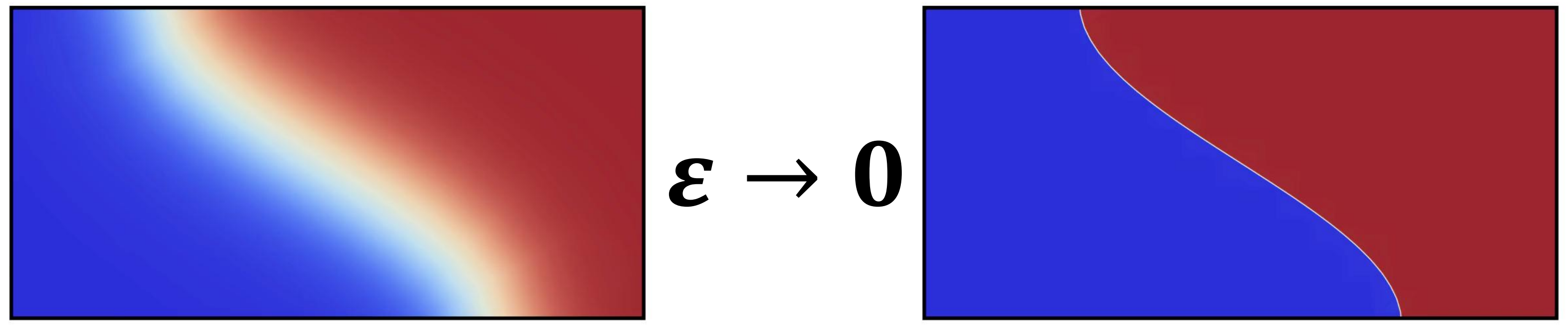


Phase Field

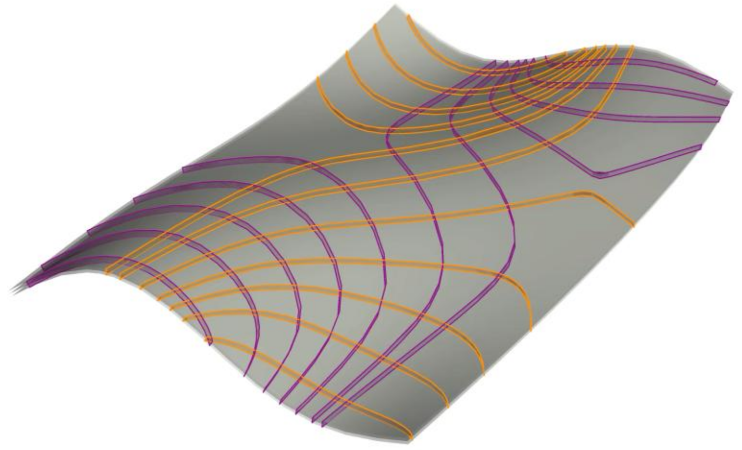


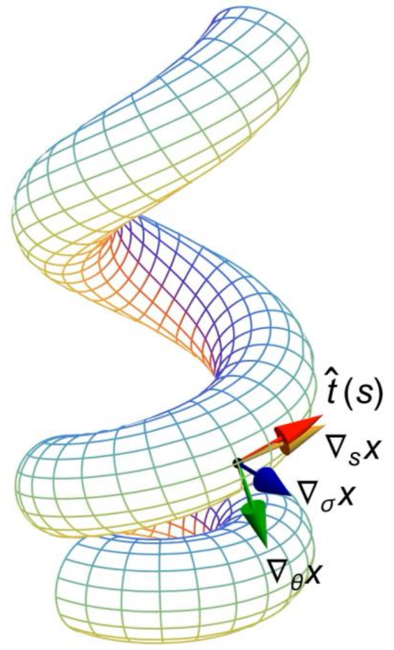
Moving Boundary

How to understand $\varepsilon \to 0$?

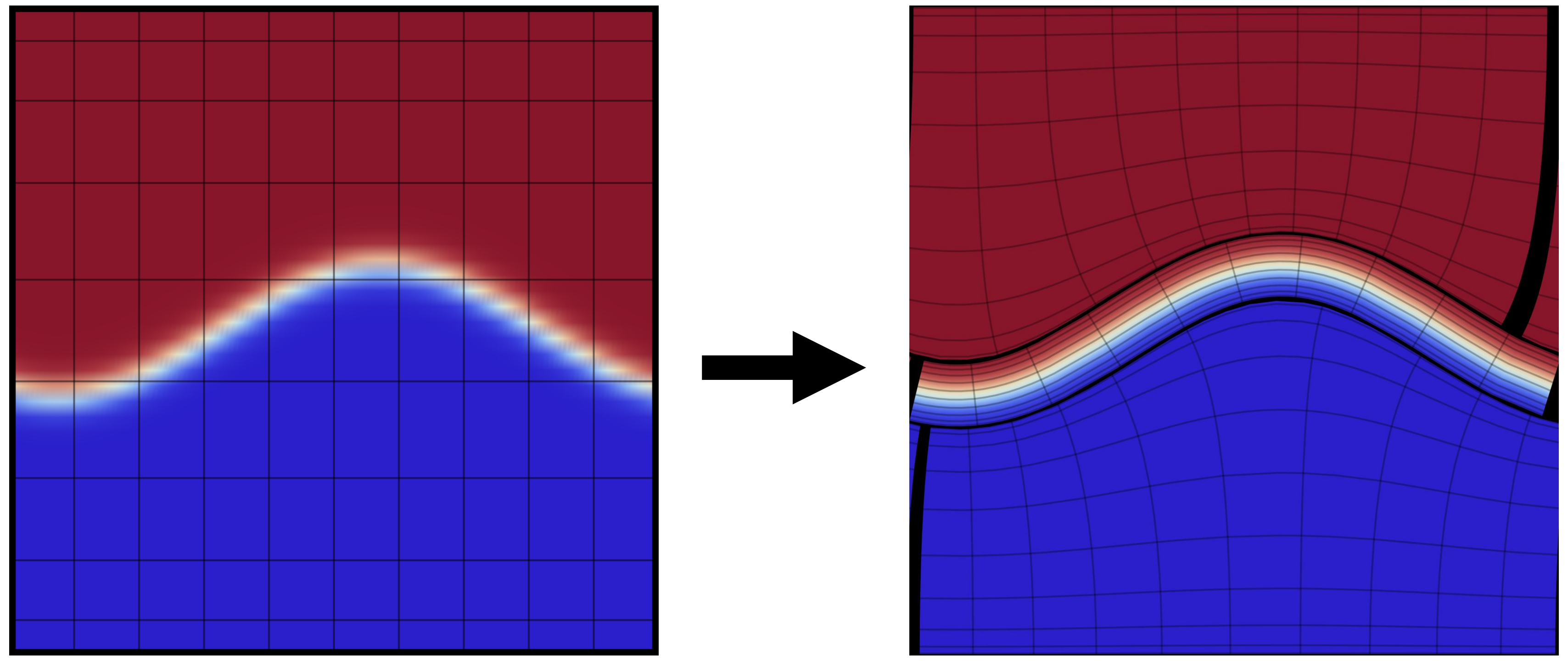


Differential geometry

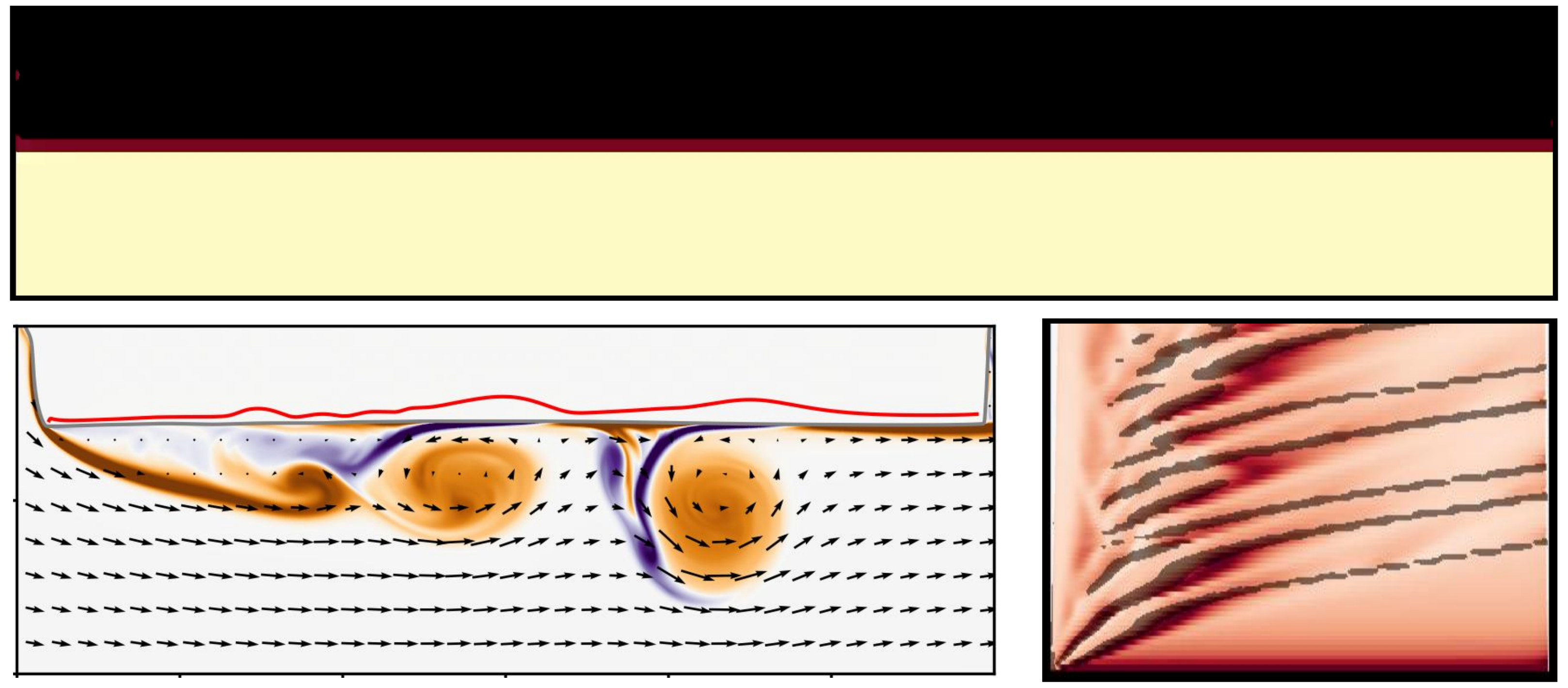




Use theory for better numerics

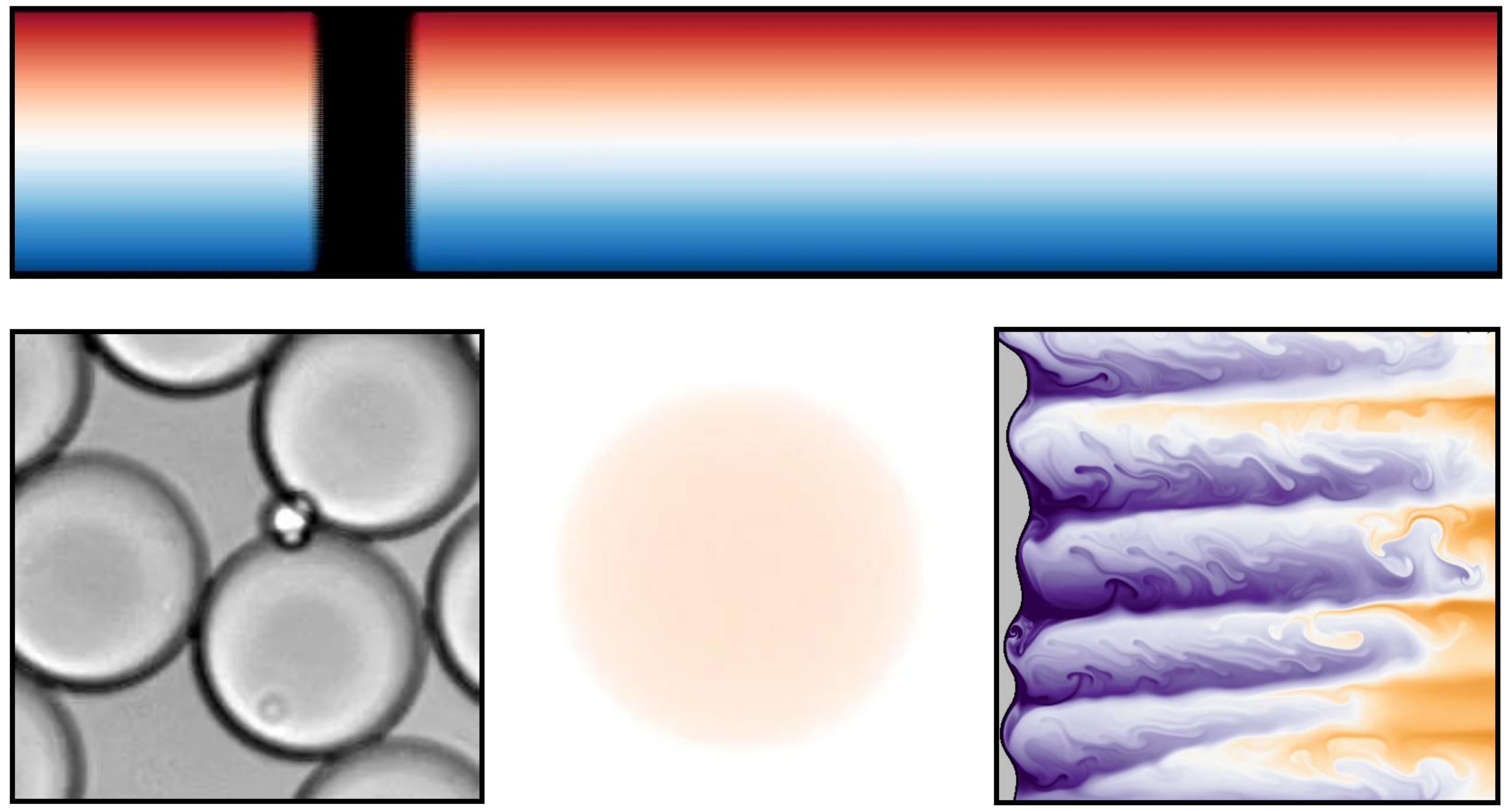


Simulations reproduce melting!



Goal: Better methods

⇒ Better science!



Eric Hester +

Phil Trinh
Chris Budd

Andrea Bertozzi
Dino di Carlo

Geoff Vasil
Keaton Burns

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Craig McConnochie

Benjamin Favier
Louis Couston

Bishak Gayen
Cat Vreugdenhil

Jon Aurnou
Cy David