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The orbit method in representation theory

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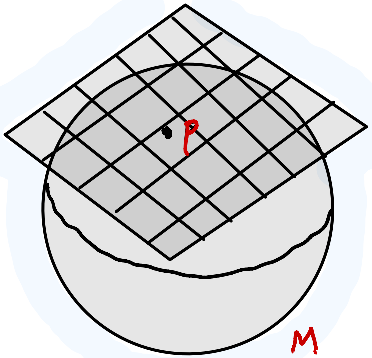
Manifolds

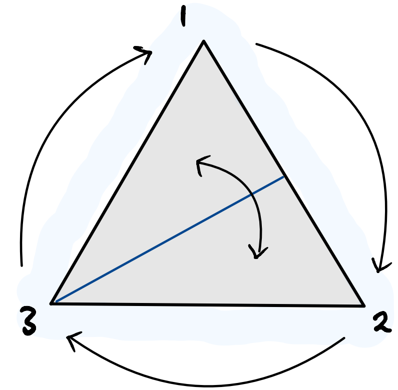
A manifold is a geometric space which locally looks like a real n-dimensional space.

To a point p on a manifold M we can attach the tangent space to M at p.

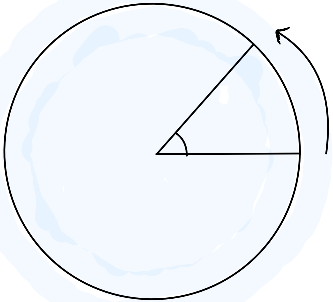


Lie groups

A group is the fundamental expression of symmetry in mathematics.



A Lie group is a manifold with the compatible structure of a group.

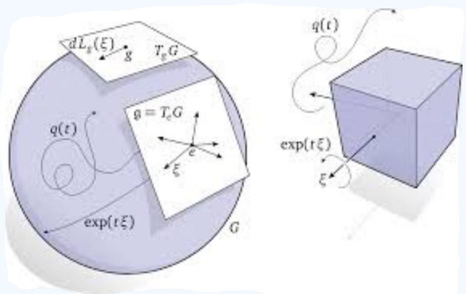


Representations

A representation of a Lie group G is a way of expressing such a group as a family of matrices.

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \\ c_7 & c_8 & c_9 \end{bmatrix}$$

Lie algebras

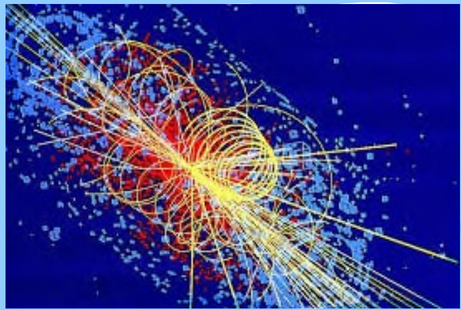


The tangent space of a Lie group inherits the structure of a Lie algebra.

Theorem: the representations of the Lie algebra are the same as those of the Lie group.

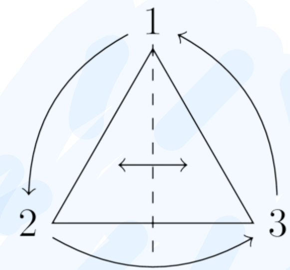
Applications of the representation theory of Lie algebras

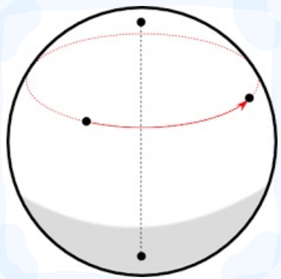
- Particle physics.
- Quantum mechanics.
- Molecular chemistry.
- the geometric Langlands program.
- ...



Orbits

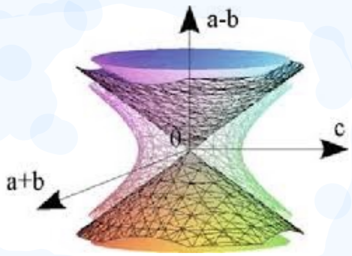
A Lie group may act on another geometric space, moving the points around in a way which respects the geometry.





The orbits of the action are the geometric subspaces consisting of points which can be permuted by the group.

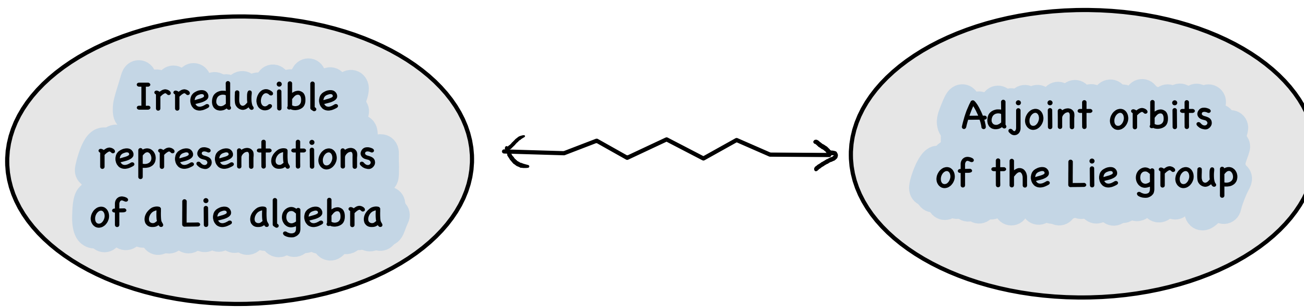
There is a very special representation, called the coadjoint, and the orbits inside it are called coadjoint orbits.



Kirillov's orbit method

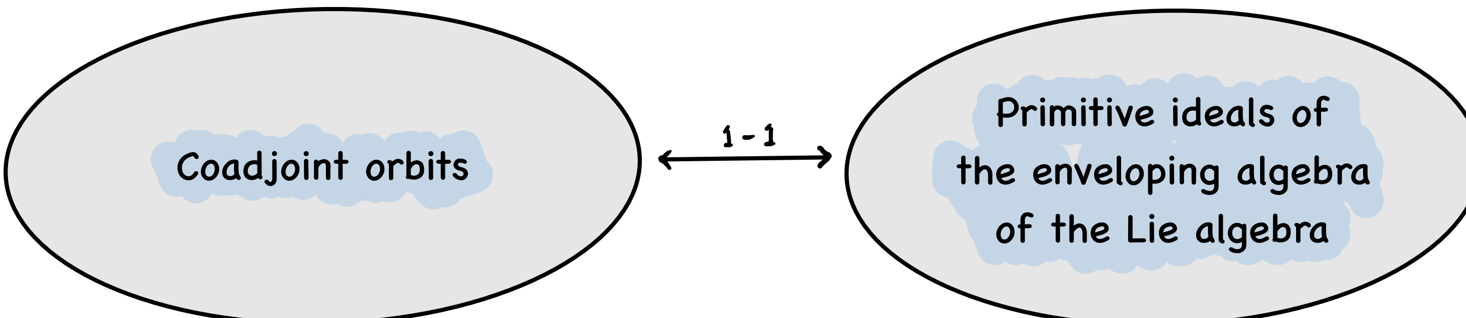
This philosophy began in the 1960s, and is retrospectively viewed as an attempt to marry symplectic geometry, representation theory and mathematical physics. Today new results on this theme are rare as hens' teeth.

A A Kirillov Jr



Losev's orbit method conjecture

Ivan Losev (Yale) breathed new life in the the core problems in the orbit method using new methods from the theory of symplectic singularities. He made a profound construction and one conjecture.



My recent breakthrough

- I spent 7 years building the methods to solve this problem.
- I proved Losev's conjecture, using various new methods from algebraic geometry and combinatorics (Dirac reduction).
- I sole-authored a paper in one of the best journals in Pure Mathematics.
- I didn't listen to the people who told me it couldn't be done.

What next?

- Generalising this work with my PhD student Lukas Tappenier. Pushing each new method as far as it can go.

- Using this breakthrough as a platform to build new collaborations and grant proposals.

- Working to expand my team and attack more profound conjectures from mathematical physics.

The End