Strategic Financing Decisions in a Spatial Model of Product Market Competition.

Richard Fairchild and Sasanee Lovisuth
University of Bath
School of Management
Working Paper Series
2005.10

This working paper is produced for discussion purposes only. The papers are expected to be published in due course, in revised form and should not be quoted without the author’s permission.
<table>
<thead>
<tr>
<th>Year</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>Specific Human Capital as an Additional Reason for Profit Sharing</td>
<td>Bruce A. Rayton</td>
</tr>
<tr>
<td>2005</td>
<td>Unpicking the Meaning of Value in Key Account Management</td>
<td>Catherine Pardo, Stephan C. Henneberg, Stefanos Mouzas and Peter Naudé</td>
</tr>
<tr>
<td>2005</td>
<td>Funding Gap or Leadership Gap – A Panel Discussion on Entrepreneurship and Innovation</td>
<td>Andrew Pettigrew and Stephan C. Henneberg (Editors)</td>
</tr>
<tr>
<td>2005</td>
<td>Measuring Affective Advertising: Implications of Low Attention Processing on Recall</td>
<td>Robert Heath &amp; Agnes Nairn</td>
</tr>
<tr>
<td>2005</td>
<td>Identifying the sub-components of intellectual capital: a literature review and development of measures</td>
<td>Juani Swart</td>
</tr>
<tr>
<td>2005</td>
<td>Knowledge work and new organisational forms: the HRM challenge</td>
<td>Juani Swart, John Purcell and Nick Kinnie</td>
</tr>
<tr>
<td>2005</td>
<td>Intra-organizational Connectivity and Interactivity with Intranets: The case of a Pharmaceutical Company</td>
<td>Niki Panteli, Ioanna Tsiourva and Soy Modelly</td>
</tr>
<tr>
<td>2005</td>
<td>Amalgamating strategic possibilities</td>
<td>Stefanos Mouzas, Stephan Henneberg and Peter Naudé</td>
</tr>
<tr>
<td>2005</td>
<td>Strategic Financing Decisions in a Spatial Model of Product Market Competition.</td>
<td>Richard Fairchild and Sasanee Lovisuth</td>
</tr>
</tbody>
</table>
Strategic Financing Decisions in a Spatial Model of Product Market Competition.

Richard Fairchild¹ and Sasanee Lovisuth².

Abstract

We develop a spatial competition model in order to analyse the relationship between product market competition and debt. We consider three versions of the model. That is, we analyse spatial competition with a) linear transportation costs, b) quadratic transportation costs, and c) vertical differentiation (with investment in product quality).

In our first two versions, debt is unambiguously increasing in the level of product differentiation, with the debt level increasing more sharply in the quadratic case. In the third case, debt is positively related to investment in product quality.

¹ School of Management, University of Bath, Claverton Down, Bath BA2 7AY, U.K., E-mail mnsrf@bath.ac.uk
² School of Management, University of Bath, Claverton Down, Bath BA2 7AY, U.K., E-mail mnmsl@bath.ac.uk
Strategic Financing Decisions in a Spatial Model of Product Market Competition.

1. Introduction

Financial and Industrial economists have increasingly recognised the interaction between product market competition and financing decisions of firms. A firm may use financial leverage strategically to affect a rival’s behaviour. There are two main modelling approaches; limited liability models, and deep purse, or predation models. In limited liability models, rival firms have an incentive to increase debt in order to soften Bertrand price competition and increase firm value (eg Showalter 1995). In predation models (eg Brander and Lewis 1986, Bolton and Scharfstein 1990, Fudenberg and Tirole 1986), a highly-leveraged firm is subject to predatory threat by a low-leveraged firm. Therefore, firms have an incentive to reduce debt levels. Hence, the limited liability and predation models provide opposite predictions. The limited liability model predicts a positive relationship between market power and debt, while the predation model predicts a negative relationship.


In recognition of these conflicting theoretical and empirical results, Fairchild (2004a and 2004b) presents a new theoretical approach which analyses the effects of the degree of market power on equilibrium debt levels. He bases his model on the analysis of Dasgupta and Titman (1998), in which long-term debt softens price competition by inducing the rivals to focus on short-term pricing decisions.

Employing a non-spatial model, Fairchild (2004a and 2004b) demonstrates that a non-linear relationship may exist, in which, for high levels of market power and product differentiation, the limited liability effect dominates, while for low levels of market power and low differentiation, the predation effect dominates. Hence, his model supports Pandey’s (2000) empirical findings.

In this paper, we develop Fairchild’s analysis by analysing the relationship between product market competition and debt in a spatial competition setting. Using transportation costs as a
measure of product market differentiation, our model enables us to examine the interaction between differentiation, consumer preferences, and debt. We consider three variants of the model. In the first version, consumers have linear transportation costs a la Hotelling (1929). In the second variant, consumers have quadratic transportation costs a la D’aspremont et al (1979). In the final version of our model, the firms are horizontally differentiated, but can choose to invest in enhancing the quality of their products in an attempt to differentiate vertically.

The main findings of our model are in sharp contrast to Fairchild’s (2004) results. We demonstrate an unambiguous positive relationship between product differentiation and debt. Hence, the predation effect always dominates the limited liability effect. However, when consumer transportation costs are quadratic, the equilibrium debt level increases more sharply than in the case of linear transportation costs. Finally, investment in vertical differentiation is affected by the level of investment costs. If investment costs are low, both firms invest in product quality. If investment costs are high, neither firm invests. In both of these cases, the firms set the same debt levels. However, if investment costs are at a medium level, only one firm invests, and differentiates itself vertically from its rival. The high quality firm then chooses a higher debt level than the low quality firm. Hence, our model predicts a positive relationship between product quality and leverage.

2. The Model

We consider a financial structure/product pricing game played in a Hotelling spatial location model. We take the duopoly firms’ locations as exogenously given. At date 0, the rivals simultaneously choose debt levels. At date 1, they observe each other’s debt choices, and simultaneously set product prices. They then compete over two periods (date 1 and date 2) in a Hotelling product market.

We build on the model of Dasgupta and Titman (1998). In deciding on their date 1 prices, the firms face the following trade-off. The short-term date 1 price affects the firms’ market shares. Assuming that there is a certain customer ‘stickiness’ (customers buying from one firm in date 1 tend to buy from the same firm in date 2), the date 1 price affects date 1 and date 2 profits. The higher the date 1 price, the higher the date 1 profits, but also the lower the market share, and the lower the date 2 profits. In addition, the firms face price competition
from each other. In summary, there are two forces driving price down; Bertrand price competition, and the firms’ desire for long-term market share. Increasing long-term debt softens price competition by inducing firms to focus on short-term pricing and profits. Since they are less interested in future market share, they compete less aggressively in short-term prices.

The timeline of the model is as follows.

**Date 0**: The duopoly firms simultaneously choose debt levels in order to maximise firm value (given each firm’s expectation of the rival’s choice of debt).

**Date 1**: The firms observe each other’s debt levels, and then simultaneously set product prices to maximise equity value (given each firm’s expectation of the rival’s choice of price). Customers observe product prices, and decide which firm to buy from (with each customer buying one unit of product from one firm or the other).

**Date 2**: The customers buy from the same firms in date 2.

The details of the model are as follows. We consider a Hotelling spatial competition model consisting of two firms, \( i \in \{A, B\} \), located exogenously in the unit interval, and a continuum of customers, located uniformly across the unit interval. Specifically, firm A is located at point \( a \) units of distance from 0, and B is located at point \( b \) units of distance from 1, with \( a = b \) (that is, the firms are symmetrically located). This is demonstrated below.

| 0 | a | x | b | 1 |

Consumers must travel to a firm to buy a product. Each consumer’s utility from buying one unit of product consists of his reservation utility \( U \) minus the price of the good \( p_i \) minus the transportation costs \( t \) per unit of distance travelled.

**Case 1: Linear Transportation Costs.**
In case 1, consumers have linear transportation costs. Consumer $X$ is the marginal consumer, who is indifferent between buying from firm $A$ and firm $B$. Consumer $X$’s utility from buying from firm $A$ or firm $B$ is

$$U_a = U - P_a - t(X - a), \quad (1)$$

$$U_b = U - P_b - t(1 - b - X), \quad (2)$$

respectively.

Since consumer $X$ is indifferent between buying from firm $A$ or firm $B$, all consumers to the left of $X$ strictly prefer to buy from $A$, while all consumers to the right strictly prefer to buy from $B$. Hence, we derive firm $A$’s date 1 demand function by equating consumer $X$’s utilities; $U_a = U_b$.

Therefore, the demand functions for firm $A$ and firm $B$ are

$$X_a = \frac{P_b - P_a}{2t} + \frac{1}{2}, \quad (3)$$

$$X_b = 1 - X_a = \frac{P_a - P_b}{2t} + \frac{1}{2}. \quad (4)$$

Following Dasgupta and Titman (1998), the firms choose their date 0 debt levels to maximise total firm value, and set their date 1 prices to maximise equity value (given their expectation of their rival’s debt and pricing decisions).

In date 2, the firms have local monopolies (due to customer stickiness). Therefore, each firm can charge its customers their date 2 reservation prices. We assume that, at date 0 and date 1, there is uncertainty about the date 2 reservation prices. Specifically, the date 2 reservation prices are drawn from a uniform distribution, as follows; $\tilde{R} \in u(0, \hat{R})$. The date 0 expectation of the reservation value is given by $E(\tilde{R}) = \int_0^{\hat{R}} \tilde{R} \partial F(\tilde{R}) = \frac{\hat{R}}{2}$.

This uncertainty is resolved at date 2, with all consumers drawing the same reservation price. Therefore, firm $A$’s and firm $B$’s date 0 expected firm values are
respectively.

If firm $A$ and firm $B$ choose respective date 0 debt levels $d_a$ and $d_b$, their respective date 0 equity values are

$$\Pi_a = p_a X_a + X_a \int_{d_a}^{\hat{R}} \hat{R} \partial \hat{F}(\hat{R}) = p_a X_a + X_a [\frac{\hat{R}^2 - d_a^2}{2\hat{R}}],$$

(7)

$$\Pi_b = p_b X_b + X_b \int_{d_b}^{\hat{R}} \hat{R} \partial \hat{F}(\hat{R}) = p_b X_b + X_b [\frac{\hat{R}^2 - d_b^2}{2\hat{R}}].$$

(8)

We solve the game by backward induction. Firstly, we solve for the date 1 prices. We take as given that firm $A$ and firm $B$ have chosen date 0 date levels $d_a$ and $d_b$. They simultaneously choose their date 1 prices to maximise equity values (equations (7) and (8)), given their expectation of their rival’s pricing decision.

We substitute (3) into (7) and (4) into (8) to obtain the equilibrium prices $P_a^*$ and $P_b^*$ by solving $\partial \Pi_a / \partial P_a = 0$, and $\partial \Pi_b / \partial P_b = 0$. Hence, we obtain the following;

**Lemma 1**: The equilibrium date 1 prices are

$$P_a^* = t - \frac{3\hat{R}^2 - d_b^2 - 2d_a^2}{6\hat{R}}.$$

$$P_b^* = t - \frac{3\hat{R}^2 - d_a^2 - 2d_b^2}{6\hat{R}}.$$
We now move back to date 0 to solve for the equilibrium debt levels. We do so by substituting \( P_a^* \) and \( P_b^* \) from lemma 1, as well as (3) and (4), into (5) and (6), and solving \( \frac{\partial V_a}{\partial d_a} = 0 \), and \( \frac{\partial V_b}{\partial d_b} = 0 \). Hence, we obtain the following.

**Proposition 1:** The equilibrium date 0 debt levels are \( d_a^* = d_b^* = \sqrt{11.33t - R + R^2} \).

Therefore, the equilibrium date 1 prices are \( P_a^* = P_b^* = t - \frac{1}{2} + \frac{34t}{6R} \).

The equilibrium date 0 firm values are \( V_a^* = V_b^* = \frac{t}{2} - \frac{1}{4} + \frac{17t}{6R} + \frac{R}{4} \).

Higher consumer transportation costs \( t \) represent a higher degree of product differentiation. Hence, proposition 1 demonstrates that the equilibrium debt level is increasing monotonically in the degree of differentiation. Conversely, as transportation costs reduce, the rivals face more intense competition, and therefore reduce debt. Hence, the predation effect dominates; firms respond to an increase in competition by reducing debt in order to reduce the threat of predation.

Note the contrast with Fairchild’s (2004) non-spatial model, where there was an inverted U-shaped relationship between market power and debt (since, in that model, the limited liability effect of debt dominated over low ranges of competition, while the predation effect dominated over high levels of competition).

Further, proposition 1 demonstrates that equilibrium prices and equilibrium firm values are unambiguously increasing (decreasing) in product differentiation (competition).

**Case 2: Quadratic transportation costs**

In case 2, consumers have quadratic transportation costs. Therefore, marginal consumer \( X's \) utility from buying from firm \( A \) or firm \( B \) is

\[
U_a = U - P_a - t(X - a)^2, \quad (9)
\]

\[
U_b = U - P_b - t(1 - b - X)^2, \quad (10)
\]
respectively.

We solve the game using exactly the same process as before. That is, we equate consumer $X$’s utility functions, $U_a = U_b$, in order to derive firm $A$’s and firm $B$’s demand functions. Then we solve for the equilibrium date 0 debt levels and date 1 prices by backward induction (Recall that the firms set their date 0 debt levels to maximise firm value, while they set their date 1 prices to maximise equity value, given their expectation of their rivals debt and pricing decisions).

We now obtain;

**Proposition 2:** The equilibrium date 0 debt levels are $d_a^* = d_b^* = \sqrt{2Rt}$. Therefore, the equilibrium date 1 prices are $P_a^* = P_b^* = \frac{4t - R}{2}$. The equilibrium date 0 firm values are $V_a^* = V_b^* = t$.

As in the case of linear transportation costs, the equilibrium debt levels are increasing in the transportation costs (that is, debt is increasing in product differentiation, and conversely, debt is reducing in competition intensity). Therefore, the predation effect again dominates. The following graph compares the rate of increase of debt in the two transportation cost cases.
The steeper line represents the effect of transportation costs on debt in the quadratic case, while the flatter line represents the effect of these costs on debt in the linear case. Note that, in the quadratic case, zero transportation costs are associated with zero debt, while in the linear case, debt is positive for all levels of transportation cost. This diagram demonstrates the interaction of consumer preferences and competition on debt levels. In the quadratic case, as transportation costs reduce, the threat of predation increases more sharply, and therefore the debt level reduces at a greater rate than in the linear case.

**Case 3: Hybrid case.**

We term case 3 our hybrid case. In this case, firms $A$ and $B$ are horizontally differentiated as before. However, each firm now chooses to invest an amount $c_i \in \{0, C_i\}$ in enhancing the features of the product. Hence, this represents an attempt to vertically differentiate the product through enhanced product quality. Of course, vertical differentiation only occurs if only one firm invests in product enhancement.

The timeline is;
Date 0: The rivals simultaneously decide whether to invest in product-enhancement. Each firm makes this decision to maximise firm value, given its expectation of the rival’s investment choice.

Date 1: The firms simultaneously choose debt levels to maximise firm value, given expectation of the rival’s choice of debt.

Date 2: The firm simultaneously choose prices to maximise equity value, given expectation of the rival’s choice of price.

Date 3 and 4: the firms trade in the product market as previously.

The marginal consumer $X$’s utility from buying from firm $A$ or firm $B$ is

$$U_a = U + \Delta_a - P_a - t(X - a),$$  
$$U_b = U + \Delta_b - P_b - t(1 - b - X),$$

where $\Delta_i$ represents the consumer’s enhanced utility from buying from firm $i$. When $c_i = 0$, $\Delta_i = 0$. When $c_i = C_i$, $\Delta_i = \delta_i$.

Having made their date 0 quality-investment decisions, the firms make financial and price decisions in period 1 and 2, respectively.

Solving $U_a = U_b$, we obtain the demand functions of A and B as follows:

$$X_a = \frac{1}{2} + \frac{P_b - P_a + \delta_a - \delta_b}{2t},$$  
$$X_b = \frac{1}{2} + \frac{P_a - P_b + \delta_b - \delta_a}{2t}.$$

We solve the model by backward induction. The firms choose date 2 prices to maximise equity values, given by (7) and (8).
Substituting for $X_a$ and $X_b$ from (13) and (14), and solving $\partial \Pi_a / \partial P_a = 0$ and $\partial \Pi_b / \partial P_b = 0$, we obtain the following;

**Lemma 2**: The equilibrium date 2 prices are

$$P_a^* = t + \frac{\delta_a - \delta_b}{3} - \frac{3R^2 - d_b^2 - 2d_a^2}{6R},$$

$$P_b^* = t + \frac{\delta_a - \delta_b}{3} - \frac{3R^2 - d_a^2 - 2d_b^2}{6R}.$$

We now move back to the date 1 financing decision. The rivals choose debt to maximise firm values, given by (5) and (6), given the date 0 utility-enhancing investment, $C$.

We substitute the prices from lemma 2 into (13) and (14), and into (5) and (6) to obtain firm values. We then solve $\partial V_a / \partial d_a = 0$, and $\partial V_b / \partial d_b = 0$, to obtain the equilibrium debt levels, given date 0 utility enhancing investments. We obtain the following;

**Lemma 3**:

1. If both firms invest in utility enhancement ($C_a = C_b = c$), or neither firm invests in utility enhancement ($C_a = C_b = 0$), the equilibrium debt levels are

   $$d_a^* = d_b^* = \sqrt{2Rt}.$$

2. If only firm $a$ invests in utility enhancement ($C_a = c, C_b = 0$), the equilibrium debt levels are

   $$d_a^* = \sqrt{2Rt + \frac{2}{5} \delta R}, \text{ and } d_b^* = \sqrt{2Rt - \frac{2}{5} \delta R}.$$

We now move back to the date 0 investment decision. Each firm makes its investment decision to maximise total firm value, given its expectation of the other firm’s investment decision.

We now substitute for the equilibrium debt levels from lemma 3 into the equilibrium prices from lemma 2, and then substitute these prices into the quantities (13) and (14). Finally, we substitute quantities and prices into (5) and (6) to obtain the following firm values. If both firms invest in product enhancement at date 1, the firm values are given by;

$$V_a = V_b = t - c. \tag{15}$$
If neither firm invests, the firm values are given by;

\[ V_a = V_b = t. \]  \hspace{1cm} (16)

If only firm \( a \) invests;

\[ V_a = t + \frac{\delta^2}{75t} + \frac{4\delta}{15} - c, \]  \hspace{1cm} (17)

\[ V_b = t + \frac{\delta^2}{75t} - \frac{4\delta}{15}. \]  \hspace{1cm} (18)

Using equations (15) – (18), we derive the Nash equilibria of the product-enhancement game, as follows;

**Proposition 3:**

\( a \)  

\[ \text{If } c \leq \frac{4\delta}{15} - \frac{\delta^2}{75t}, \text{ both firms invest in utility enhancement } (C_a = C_b = c). \text{ Therefore, the equilibrium debt level is } d_a^* = d_b^* = \sqrt{2Rt}, \text{ and equilibrium firm values are } V_a = V_b = t - c. \]

\( b \)  

\[ \text{If } \frac{4\delta}{15} - \frac{\delta^2}{75t} < c \leq \frac{4\delta}{15} + \frac{\delta^2}{75t}, \text{ only one firm invests in utility enhancement } (C_a = c, C_b = 0). \text{ Therefore, the equilibrium debt levels are } d_a^* = \sqrt{2Rt + \frac{2}{5} \delta R}, \text{ and } d_b^* = \sqrt{2Rt - \frac{2}{5} \delta R}. \text{ The equilibrium firm values are } V_a = t + \frac{\delta^2}{75t} + \frac{4\delta}{15} - c, \text{ and } V_b = t + \frac{\delta^2}{75t} - \frac{4\delta}{15}. \]

\( c \)  

\[ \text{If } \frac{4\delta}{15} - \frac{\delta^2}{75t} < \frac{4\delta}{15} + \frac{\delta^2}{75t} < c, \text{ neither firm invests in utility enhancement. Therefore, the equilibrium debt level is } d_a^* = d_b^* = \sqrt{2Rt}, \text{ and equilibrium firm values are } V_a = V_b = t. \]
Therefore, the equilibrium of the game is affected by the product-enhancement investment costs. We are particularly interested in the effect of these costs on equilibrium debt levels. We note that, when investment costs are very low (Proposition 3a) or very high (proposition 3c), the rivals make the same investment decision, and the debt level is identical and symmetric in both cases. However, when investment costs are at a medium level (proposition 3b), only one firm invests, and we obtain asymmetric equilibrium debt levels. The firm that invests in quality is able to set a higher debt level than the firm that does not invest. The intuition is that, by investing in quality, the firm attracts more customers, and is able to charge a higher price. Therefore, this firm can set a higher debt level to further soften price competition without fear of predation.

Conclusion.

We have presented a spatial competition model in order to analyse the relationship between product market competition and rival firms' financial leverage. We compared three versions of the model; spatial competition with a) linear transportation costs, b) quadratic transportation costs, and c) vertical differentiation.

In our first two versions, the predation effect dominates over the limited liability effect (there is a monotonic positive relationship between product differentiation and debt). However, in the quadratic case, the debt level increases more sharply as differentiation increases. In the third case, debt is positively related to investment in product quality.

In this paper, we have purely considered the strategic use of debt in relation to the product market. Future research will incorporate the effect of agency costs and asymmetric information into our model.
References:


<table>
<thead>
<tr>
<th>Year</th>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004.01</td>
<td>Stephan C. M. Henneberg</td>
<td>Political Marketing Theory: Hendiadyoin or Oxymoron</td>
</tr>
<tr>
<td>2004.02</td>
<td>Yi-Ling Chen &amp; Stephan C. Henneberg</td>
<td>Political Pulling Power. Celebrity Political Endorsement and Campaign Management for the Taipei City Councillor Election 2002</td>
</tr>
<tr>
<td>2004.03</td>
<td>Stephan C. Henneberg, Stefanos Mouzas &amp; Pete Naudé</td>
<td>Network Pictures – A Concept of Managers’ Cognitive Maps in Networks</td>
</tr>
<tr>
<td>2004.05</td>
<td>Yvonne Ward &amp; Andrew Graves</td>
<td>A New Cost Management &amp; Accounting Approach For Lean Enterprises</td>
</tr>
<tr>
<td>2004.06</td>
<td>Jing Lin Duanmu &amp; Felicia Fai</td>
<td>Assessing the context, nature, and extent of MNEs’ Backward knowledge transfer to Chinese suppliers</td>
</tr>
<tr>
<td>2004.08</td>
<td>Richard Fairchild</td>
<td>Behavioral Finance in a Principal-agent Model of Capital Budgeting</td>
</tr>
<tr>
<td>2004.10</td>
<td>Stephan C. M. Henneberg</td>
<td>Operationalising a Multi-faceted Concept of Strategic Postures of Political Marketing</td>
</tr>
<tr>
<td>2004.11</td>
<td>Felicia Fai</td>
<td>Technological Diversification, its Relation to Product Diversification and the Organisation of the Firm.</td>
</tr>
<tr>
<td>2004.13</td>
<td>Bruce A. Rayton and Suwina Cheng</td>
<td>Corporate governance in the United Kingdom: changes to the regulatory template and company practice from 1998-2002</td>
</tr>
<tr>
<td>Year</td>
<td>Author</td>
<td>Title</td>
</tr>
<tr>
<td>------</td>
<td>----------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>2004</td>
<td>Bruce A. Rayton</td>
<td>Examining the interconnection of job satisfaction and organizational commitment: An application of the bivariate probit model</td>
</tr>
</tbody>
</table>